Costly state verification and general equilibrium
A note on Carlstrom and Fuerst (1998)

Jean-Marie RENAUD*

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Abstract
We extend the model of Carlstrom and Fuerst (1998) by making the cost of verification endogenous, being the result of the production of costly information incorporating capital and labor. The financial sector is then truly endogenous, the financial intermediary being the producer of information. This allows us to consider the occurrence of technological shocks on the auditing technology. Moreover, an endogenous financial intermediary makes it possible to calibrate the size of the financial sector instead of the default rate in order to give more way to the financial accelerator. We then compare our results to previous ones and reconsider whether the real business cycle methodology can support the financial accelerator view.

Résumé
Nous amendons le modèle de Carlstrom et Fuerst (1998) en introduisant des coûts de vérification endogènes, la production d’information résultant de la combinaison de capital et de travail. Le secteur financier ainsi obtenu est le producteur de l’information, ce qui nous permet de considérer la possibilité de chocs sur la technologie de production de l’information. De plus, cette endogénéité du coût de vérification nous permet de calibrer la taille du secteur financier au lieu du taux de faillite et ce afin de motiver un accélérateur financier plus important. Nous pouvons alors comparer nos résultats à ceux obtenus dans la littérature et réexaminer la capacité des cycles réels à incorporer un accélérateur financier non marginal.

* Université Paris IX Dauphine (CREFED-CERPEM)
Place du Mal de Lattre de Tassigny 75 775 Paris Cedex 16
Tel : 01-44-05-48-88 Fax : 01-44-05-40-98 Mail : Jean-Marie.Renaud@dauphine.fr
1 Introduction

In a series of seminal papers, Carlstrom and Fuerst have embedded a costly state verification problem in the Real Business Cycles framework. The motivation for such a research program is to motivate a non-trivial role for financial factors in the Business Cycle.

Fuerst (1995) is the first of these papers. The investment and consumption goods are no longer homogeneous. The technology transforming the consumption good in an investment good is held by entrepreneurs living a single period of time. This intra-periodic linear technology incurs an idiosyncratic risk and its outcome is observed with no cost only from its owner. The entrepreneurs supply their labor inelastically and the wage thus earned constitutes their sole wealth. External financing being necessary, entrepreneurs borrow from financial intermediaries who prevent the duplication of auditing costs. The optimal contract is standard risky debt and entails a financial constraint since the level of investment is tied down by the entrepreneurs’ level of wealth. A financial accelerator is thus obtained: a positive technological shock raises the wealth of entrepreneurs and allows higher levels of investment. A comparison with the standard Real Business Cycles model reveals the weakness of this effect. The immediate response of investment to a technological shock is indeed dampened by the financial constraint as the rise in entrepreneurs’ wealth is insufficient to allow the optimal increase in investment. The propagation of the shock is on the contrary feebly improved by the financial constraint as investment stays above its steady state level for a longer period of time. The results of Fuerst (1995) thus show that the financial accelerator does very little to augment the propagation of the standard Real Business Cycles model, the amplification being slightly reduced.

Carlstrom and Fuerst (1997) introduce infinitely-lived entrepreneurs in order to enrich the dynamic of entrepreneurial wealth. With infinitely-lived entrepreneurs, the level of wealth depends not only on the current income but also on past savings. This hypothesis raises two delicate issues. First of all, one may wonder whether long term contracting could not mitigate or even solve the asymmetric information problem. The authors choose to ignore this issue by restraining their analysis to intra-periodic contracts. Even so, the asymmetric information issue would not be enduring if entrepreneurs were to accumulate enough wealth to ensure self-financing. This problem is easily circumvented by imposing a higher discount rate for entrepreneurs (as compared to consumers). Under these hypotheses, the model exhibits properties of some interest but remains disappointingly close to the standard Real Business Cycles model. As in Fuerst (1995), the amplification remains
dampened by the financial constraint which prevents an immediate optimal response of investment to a productivity shock. Nevertheless, the dynamics of entrepreneurial wealth is responsible for a hump-shaped response of output, relevant according to Cogley and Nason (1995). Indeed, entrepreneurs contemporaneously increase their saving which allows a further increase of investment and production in the periods following the shock. Overall, the dynamic properties of the model differ little from the ones of the standard Real Business Cycles model.

Carlstrom and Fuerst (1998) (hereafter CF) compare the results of the previous model with those obtained when the financial constraint lies upon the production of the consumption good. The expected result is to have a stronger financial accelerator by imposing a financial constraint to the entire economy. On the contrary, the authors obtain a weaker financial accelerator, such a constraint having weaker distortionary effects.

The costly state verification framework thus provides little support for the financial accelerator view within a Real Business Cycles model. Two major criticisms come readily to the mind. First, the infinite horizon hypothesis can be seen as a major obstacle to enduring market failures. Second, bankruptcies as they are measured in modern economies reveal themselves far too scarce to give rise to any notable financial accelerator effect. Hence, the models under discussion are unable of displaying fluctuations notably different from the standard Real Business Cycles model.

This paper deals with the second of these criticisms. We argue that the default rate and the auditing costs of such a normative model should be understood in a very wide way. To support this view, we define the cost of observing the outcome as resulting from the production of costly information and not as a loss of output. The partial equilibrium approach of Townsend (1979) uses the latter hypothesis as a way of simplifying the analysis. Yet, in a general equilibrium model, it does not seem unwise to consider that what is lost in the course of auditing is not a loss to everyone in the economy. Thus, the auditing cost stems from the need to produce information and this production is not different from the production of any other good, i.e. it requires the use of inputs, capital and labor. Once this view is adopted, one can implement an alternative calibration resting on the size of the banking sector and allowing for higher default rates.

The following section describes the Townsend (1979) costly state verification problem. The third section presents our modified version of the CF model including endogenous monitoring costs. Our calibration and our results are the object of the fourth section and a final section offers a conclusion.
2 The optimal financial arrangement

We present in this section the intra-periodic financial scheme under asymmetric information. An entrepreneur operating a risky technology with insufficient wealth borrows from a bank confronted with a costly state verification problem.

2.1 The entrepreneur

Entrepreneur $i$ is infinitely-lived, risk neutral with preferences over consumption and operates a firm producing the consumption good:

$$y_{it} = \theta_t \omega_{it} F(k_{it}^y, h_{it}^y)$$  \hspace{1cm} (1)

The constant returns to scale technology is plagued with two sorts of risk. The aggregate technological shock ($\theta_t$) is public information whereas the individual idiosyncratic shock ($\omega_{it}$) is the private information of the entrepreneur. $\theta_t$ is serially autocorrelated in the usual way while $\omega_{it}$ is i.i.d across firms and time, log-normally distributed (with density $\phi$ and distribution function $\Phi$) and of unitary mean. The intra-periodic sequence of an entrepreneur’s decisions is as follows:

1. The aggregate shock is observed, the firm chooses its levels of inputs and pays for them
2. Production takes place, the firm observes its idiosyncratic shock and the output is sold
3. The entrepreneur makes his saving-consumption decision

Such a sequence of events obviously raises a financing problem. The input bill of firm $i$ is indeed

$$m_{it} = w_t h_{it}^y + r_t k_{it}^y$$  \hspace{1cm} (2)

whereas its net worth$^1$ is:

$$n_{it} = k_{it}^C (1 - \delta + r_t)$$  \hspace{1cm} (3)

The external financing required is then defined by:

$$b_{it} = m_{it} - n_{it}$$  \hspace{1cm} (4)

In a full information setting, the optimal solution would resort to contingent contracts and eliminate the idiosyncratic risk at no cost. The model would simply boil down to the standard Real Business Cycles model. Yet, with private information, this result no longer holds. External parties can observe

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$^1$ As in Carlstrom and Fuerst (1998), we neglect the fact that an entrepreneur needs another source of income in order to resume his activity after he has failed. Since this income can be arbitrarily small, it does not affect the dynamic properties of the model.
the realized output at a cost so that we are in a costly state verification framework similar to the one of Townsend (1979). In such a framework, it is well known since it was demonstrated by Gale and Hellwig (1985) that the optimal financial arrangement is the standard risky debt contract. This contract stipulates that in case of default, the creditor will audit and confiscate the entire outcome. When default does not occur, the entrepreneur pays a fixed rate of interest. The bank will charge the entrepreneur an interest rate in order to cover the expected costs associated with default. This implies that the consumption good must sell at a mark-up ($p_t$) as entrepreneurs pass this cost along to consumers.

The entrepreneur will default on his loan when his realized profit is negative, that is whenever:

$$\theta_t \omega_{it} F(k_{it}^y, h_{it}^y) - (1 + r^b_t)b_{it} < 0$$  \hspace{1cm} (5)

We can define $\omega_{it}$ as the level of the idiosyncratic shock below which the entrepreneur defaults, namely:

$$\omega_{it} = \frac{(1 + r^b_t)b_{it}}{\theta_t F(k_{it}^y, h_{it}^y)}$$  \hspace{1cm} (6)

The expected profit of entrepreneur $i$ is then:

$$\Pi_{it} = \int_{-\omega}^{\infty} \omega_{it} \theta_t F(k_{it}^y, h_{it}^y) d\Phi(\omega_{it}) - (1 - \Phi(\omega_{it}))(1 + r^b_t)b_{it}$$  \hspace{1cm} (7)

The first term on the right hand side of (7) is the product kept by the entrepreneur when he does not default while the second term is repaid to the bank under the same circumstances. In case the entrepreneur defaults, he does not keep any product nor does he pay anything to the bank.

### 2.2 The bank

As demonstrated by Williamson (1986), banks arise endogenously as a way of avoiding duplication of monitoring costs and of diversifying away the idiosyncratic risk. In contrast with CF we let the quantity of information produced in the course of auditing be a proportion $\mu$ of the size of the audited project, $m_{it}$:

$$y_{it}^b = \mu p_t m_{it}$$  \hspace{1cm} (8)

Since it is no longer the cost of auditing that is proportional to the size of the project, we note $\nu_t$ the price of the information good. We will later show how endogenous monitoring costs fit in a general equilibrium model.

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2 All equations are written in terms of ex post consumption goods, i.e, all the exchanges taking place before the idiosyncratic shock must be multiplied by the mark-up $p_t$. 

The bank’s expected profit on the loan to entrepreneur $i$ is:

$$Z_{it} = (1 - \Phi(\omega_{it})) (1 + r^b_t) b_{it} + \int_0^{\omega_{it}} \omega_{it} \theta_t F(k^u_{it}, h^y_{it}) d\Phi(\omega_{it}) - \Phi(\omega_{it}) \nu_t y^b_{it} - b_{it} \quad (9)$$

The first term on the right hand side of (9) is obtained by the bank when the entrepreneur does not default (with probability $1 - \Phi(\omega_{it})$), the second term when the entrepreneur defaults (with probability $\Phi(\omega_{it})$), the third term is the cost of observing the realized output in the latter case and the fourth term is repaid to the depositors.

The problem of the bank is to maximize the expected profit of the entrepreneur subject to the constraint that its own expected profit be non negative. Competition between banks ensures that this participation constraint is binding. For notational convenience (see Appendix A for details), we write this problem as:

$$\max \quad p_t m_{it} g_1(\omega_{it})$$

$$s.t. \quad (p_t g_2(\omega_{it}) - 1) m_{it} + n_{it} = 0 \quad (10)$$

The optimal contract is defined by the size of the project and the critical value of the idiosyncratic shock:

$$\frac{1}{p_t} = 1 - \Phi(\omega_{it}) \mu \nu_t + \phi(\omega_{it}) \mu \nu_t \frac{g_1(\omega_{it})}{g_1'(\omega_{it})} \quad (11)$$

$$m_{it} = n_{it} \frac{1}{1 - p_t g_2(\omega_{it})} \quad (12)$$

3 A general equilibrium model

The model includes three types of agents: Households, entrepreneurs and banks. We now describe each of these agents in turn and the market equilibria of our model economy. As our model closely resembles the model of CF, we will only briefly outline the model below (details and notations are given in Appendix B).

3.1 Households

The representative household is infinitely-lived and has preferences over consumption ($c_t$) and leisure ($l_t$):

$$E \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \quad (13)$$
The representative individual rents his work force and his capital in order to finance consumption and capital accumulation. The budget constraint is thus:

\[ k_{t+1}^h - (1 - \delta)k_t^h + c_t = w_t (1 - l_t) + r_t k_t^h \]  

(14)

For the lapse of time between the payment of the inputs and the sale of the output, the households deposit their income in banks. This deposit being intra-periodic, households do not demand any interest on it and the bank can repay the deposit since it eliminates the idiosyncratic risk on loans to entrepreneurs. The households are thus passive suppliers of funds to the banking sector and the financial problem does not appear in their budget constraint.

Usual first order conditions obtains:

\[ \frac{\partial u(c_t, l_t)}{\partial l_t} = \frac{\partial u(c_t, l_t)}{\partial c_t} w_t \]  

(15)

\[ \frac{\partial u(c_t, l_t)}{\partial c_t} = E \left[ \beta \frac{\partial u(c_{t+1}, l_{t+1})}{\partial c_{t+1}} (1 - \delta + r_{t+1}) \right] \]  

(16)

3.2 Entrepreneurs

The economy contains a continuum of entrepreneurs with unit mass. The constant returns to scale on both the production of the consumption good and of information allows easy aggregation. At the end of the period, the representative entrepreneur makes a saving-consumption arbitrage. The aggregate profit of the entrepreneur available for this purpose can be written as:

\[ \Pi_t = \theta_t F(k_t^y, h_t^y) \left( \int_{\omega_t}^\infty \omega_t d\Phi (\omega_t) - (1 - \Phi (\omega_t)) \omega_t \right) \]  

(17)

Aggregating across banks, one can obtain the zero profit condition of the banking sector which can be used to write (17) as:

\[ \Pi_t = n_t + \theta_t F(k_t^y, h_t^y) \left( 1 - \Phi (\omega_t) \mu \nu - \frac{1}{p_t} \right) \]  

(18)

Hence, the problem of the entrepreneur is to choose \( c_t^e \) and \( k_t^e \) in order to

\[ \max \ E \sum_{t=0}^\infty (\beta \xi)^t c_t^e \]

s.t. \( k_{t+1}^e + c_t^e = (1 - \delta + r_t) k_t^e + y_t \left( 1 - \Phi (\omega_t) \mu \nu - \frac{1}{p_t} \right) \)  

(19)

\( \xi \) is introduced so that entrepreneurs discount the future more heavily than household. Without this hypothesis, entrepreneurs would simply accumulate enough wealth to ensure self financing and
escape the asymmetric information problem. Because of risk neutrality, the first order condition equates the expected rate of return on internal funds times the discount factor to unity:

$$1 = E \left[ \beta^t \frac{1 + g_1(\omega_t)}{1 - r_{t+1}} \right]$$  

(20)

The pricing of inputs includes the mark-up so that factor prices are below their marginal productivity in order to pay for the auditing costs:

$$\theta_t \frac{\partial F(k_t, h_t)}{\partial k_t} = p_t r_t$$  

(21)

$$\theta_t \frac{\partial F(k_t, h_t)}{\partial h_t} = p_t w_t$$  

(22)

3.3 Banks

We have argued in the introduction that the monitoring costs should, in a general equilibrium analysis, be endogenous. We thus do not consider the cost of auditing the project to be a loss of output to the economy but rather that it is the result of the production of costly information. The representative bank operates a constant returns to scale technology:

$$y_t^b = \theta_t^b F^b(k_t^b, h_t^b)$$  

(23)

where $\theta_t^b$ is the autocorrelated productivity shock of the financial sector.

In order to gain knowledge on the level of production, the bank must therefore rent capital and labor so that the ex post cost associated with an input bill ($m_t$) is:

$$\nu_t y_t^b = p_t (r_t k_t^b + w_t h_t^b)$$  

(24)

While the inputs that the bank will need to evaluate a given project are not known when the markets for inputs take place, aggregating across projects allows the bank to participate in those markets. Averaging out, the bank indeed knows with certainty the inputs it will need.

Competition among banks drives the profit of the banking sector to zero. The pricing of inputs thus equates the price of the input to the marginal productivity of the input in the banking sector in terms of the final output:

$$\nu_t \theta_t^b \frac{\partial F^b(k_t^b, h_t^b)}{\partial k_t^b} = p_t r_t$$  

(25)

$$\nu_t \theta_t^b \frac{\partial F^b(k_t^b, h_t^b)}{\partial h_t^b} = p_t w_t$$  

(26)
where \( \nu_t \) is the price of information in terms of the final output. The input prices are below their marginal productivity since the market for inputs take place before the idiosyncratic shocks are observed.

### 3.4 Market equilibria

The market equilibria for labor, capital, information and the consumption good can be stated as:

\[
\begin{align*}
    h_t &= h^y_t + h^b_t \tag{27} \\
    k^c_t + k^b_t &= k^y_t + k^b_t \tag{28} \\
    y^b_t &= \Phi(\omega_t) \mu_p m_t \tag{29} \\
    y_t &= c + c^e_t + i_t \tag{30}
\end{align*}
\]

### 4 The financial accelerator in a Real Business Cycles model

#### 4.1 Calibration

We choose not to calibrate our model in order to match the bankruptcy rates as measured by prior empirical studies. It has been previously argued in the introduction that default in such a normative model does not correspond to the legal procedure of bankruptcy. Default should instead be understood to occur whenever a bank needs to audit a particular project after a client has declared that the project had not yielded the expected result. A first method could be to consider that the sole activity of the banking sector is to audit the projects that fail. We could then simply calibrate the model in order to match the percentage of hours worked in the banking sector. This appears to be a somewhat extreme view as banks have other activities and we thus retain a steady state share of employment in the banking sector of 2%. This figure can be compared to the 10% of Cudeville and Hairault (1998), a model where the sole purpose of the banking sector is to offer transaction services. The unobservable parameter \( \xi \) is set accordingly and is responsible for a very high default rate (over 13% at the steady state). The rest of our calibration, given in table 1, is standard and follows CF for the sake of comparison.

The comparison of the steady state obtained with various calibration of our model with the one of CF yields useful insights (see table in appendix C). When computed with the full calibration of
Table 1. Calibration

<table>
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<tr>
<th>(\delta)</th>
<th>(\beta)</th>
<th>(\alpha)</th>
<th>(\alpha^b)</th>
<th>(l)</th>
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<th>(\rho^b)</th>
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CF, the steady state of our model mimics almost perfectly the steady state of CF. This occurs because the agency problem is of a limited magnitude and the probability of failure is calibrated. Yet, the input bill in our model is smaller since entrepreneurs do not pay for the inputs used in the process of auditing. Hence, the net wealth necessary to achieve the desired level of failure is lower in our model calibrated after CF than in the CF model. Our model does not differ notably from CF but simply allows a description of the financial sector of a very small size (employing less than 0.15% of the labor force) when the calibration of CF is retained. We have chosen to increase the size of this sector by increasing the severity of the agency problem. This is easily done by restraining the saving of entrepreneurs who default more often, their net wealth being weaker. The cost of increasing the size of the banking sector is a rapid rise in the probability of failure and therefore of the risk premium (e.g. one third of the projects fail when the ratio of employment in the banking sector to employment in the consumption good sector is calibrated to 5%).

4.2 Impulse response functions

The purpose of our modified version of CF is to increase the potency of the financial accelerator. Indeed, monitoring costs in the CF model only feebly improve the propagation of a productivity shock on the consumption good technology while reducing amplification. Our endogenous monitoring costs hypothesis allows us to use an alternative calibration by which the agency problem is greatly strengthened. Moreover, we can study the impact of a productivity shock on the auditing technology. We thus compute the impulse-response functions to the two productivity shocks, which are plotted in appendix D.

A productivity shock on the consumption good technology  We first describe the forces at work when a productivity shock occurs on the consumption good technology. We then compare our results with those of the standard Real Business Cycles and those of CF. While doing so, one must be careful to compare relevant variables from the different models. Hence, we compute the aggregate output and the aggregate consumption of our model and compare our aggregate output with the output available after the auditing costs in CF.
A higher productivity of the consumption good technology stimulates the demand for inputs. Hence, labor and investment rise on impact. Investment on the part of entrepreneurs is further motivated by the need to increase their wealth in order to finance a higher level of production. Yet, since the wealth of entrepreneurs consists of previously accumulated capital, the increase of production is constrained in the first period and is responsible for a rise of the default rate and consequently of the risk premium and the mark-up. After entrepreneurs have accumulated enough wealth to finance a higher level of production, the default rate, the risk premium and the mark-up all fall below their steady state value.

The sector producing the consumption good (hereafter sector one) must, in the first period, compete with the banking sector (hereafter sector two) on the market for inputs. Thus, the higher default rates limit the increase of hours worked in sector one while capital is partially reallocated to sector two. Hence, the hump-shape response of output in sector one is dampened when one considers aggregate output by the strong rise on impact of output in sector two. Once the financing capacity of entrepreneurs has been improved, output of sector two falls below its steady state while capital is reallocated to the more productive sector one. The behavior of households is a standard response to an autocorrelated productivity shock: they increase their consumption and saving thanks to a rise in their labor supply.

The qualitative process at work in our model does not differ notably from CF. Moreover, the present model’s impulse-response functions differ from those of CF chiefly because of our alternative calibration. Thus, the fall of entrepreneurial consumption is limited by the assumption of a higher level of discounting as compared to CF. From this stems the smaller rise in investment, capital, output of sector one and failure probability, among others. Nevertheless, the endogeneity of the auditing costs allows us to consider in greater detail the behavior of aggregate output and consumption. CF do not report the behavior of aggregate consumption and seemingly describe the behavior of gross output.

The distinction between gross output and output net of the auditing costs is of little relevance in the CF model since the agency problem is very weak. Thus, gross and net output basically respond in the same way to a productivity shock. Yet, in our model, the agency problem is more stringent. To consider aggregate output or output of sector one yields different results in terms of

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3 Were we to retain completely the calibration of Carlstrom and Fuerst (1998), our results would only be slightly different from theirs due to the reallocation of capital between our two sectors.
propagation and amplification. Indeed, the response of aggregate output on impact is stronger than that of the sole sector one while the peak impact (one period after the shock) of the latter is higher. The propagation thus appears to be stronger when one measures the half life of sector one output (following the definition of CF). Hence, a study of aggregate output yields a stronger amplification and a weaker propagation than the study of sector one. The endogeneity of monitoring costs, by allowing a definition of aggregate output, mitigates the conflict between amplification and propagation.

That CF do not consider the behavior of aggregate consumption is more surprising since their model yields a very strong negative response of entrepreneurial consumption to a productivity shock. This implies a negative response on impact of the aggregate consumption. Our model dampens the negative on impact response of consumption (-10% against -50% in CF) but still exhibits this awkward property at odds with stylized facts and the standard Real Business Cycles model.

**A productivity shock in the financial sector** The hypothesis of endogenous monitoring costs makes it possible to examine how an increase in the productivity of the auditing technology affects the economy. Our assumption of a quantity of information proportional to the size of the investment project plays a key role. Under the standard assumption of a proportional cost, an increased productivity of the auditing technology would not have any notable effect. The price of information would indeed be lowered but the quantity of information necessary to audit a project would augment so as to exactly off set the fall of the price. Hence, a technical progress in the financial sector would not raise aggregate output and be wholly fruitless. To avoid this pitfall, we replace the hypothesis of a proportional cost by the hypothesis of a proportional quantity of information.

A rise of productivity in sector two lowers the cost of auditing an investment project as less inputs are required and the price of information consequently falls. An higher level of production can thus be financed and the increased output of sector one stimulates the production of information. Failures indeed rise on impact as the higher production is undertaken with a level of entrepreneurial wealth inherited from the past.

Entrepreneurs do not find the situation thus created by the shock, i.e. an higher rate of default with cheaper failure, optimal. Since failure has become cheaper, the net wealth inherited from the past is too high. Hence, on impact, entrepreneurs augment their consumption and reduce their capital accumulation to achieve a lower level of net wealth. Afterwards, entrepreneurs build back their net wealth, increasing their capital stock and reducing consumption, as the productivity shock fades away.
As above, the behavior of households is a standard response to an autocorrelated productivity shock: they increase their consumption and saving thanks to a rise in their labor supply. Both factors are thus increased after the first period fall in the stock of capital due to the behavior of entrepreneurs. These resources are allocated first to sector one and then, after entrepreneurs have reduced their net wealth and as the effect of the productivity shock dwindles, also to sector two.

Our model differs from the CF model by the existence of endogenous monitoring costs and a severe agency problem. Yet, our model fails to deliver qualitative properties notably different from CF. The response to a productivity shock on the consumption good technology is halfway between the response of the standard Real Business Cycles model and that of CF. The conflict between propagation and amplification is thus reduced. Nevertheless, our model exhibits the same disturbing properties obtained by CF. The productivity shock on the auditing technology displays interesting properties but has a weak aggregate impact.

We have assumed above that the two sectors of the economy were hit by totally independent productivity shock. This has allowed us to study the qualitative response of our economy to a technological change in the auditing technology. Yet, the reverse hypothesis of a single shock appears adequate in order to compare the results of the endogenous monitoring costs model with the benchmark of standard Real Business Cycles. Indeed, the productivity shock in the latter affects the whole economy. In order to replicate this property, We consider in the remaining of the paper that the technological changes affect both sectors of the economy in the same manner. Hence, we merge the two shocks described above. The unique shock hitting the economy has a stronger impact and attenuates the disturbing properties of our model (e.g. the negative response of entrepreneurial consumption is reduced).

4.3 Dynamic properties

The quantitative properties of our model are summarized by the second order moments averaged over 100 simulations of 100 periods. We first compare the main properties of our model with Endogenous Monitoring Costs (henceforth EMC) with a unique productivity shock to the standard Real Business Cycles model (table 2) and then describe the quantitative properties of the financial sector of the EMC model (table 3). A discussion on the empirical relevance of the EMC model

4 The strong response of entrepreneurial consumption in the Carlstrom and Fuerst (1998) model prevented us from simulating their model.
The volatility of output and its components in the EMC model appears at first sight puzzling. Indeed, when compared to the standard RBC model, the EMC model displays a more volatile aggregate consumption and investment while aggregate output exhibits a lower volatility. This paradox also shows when one considers the persistence and comovement with aggregate output. Indeed, the first-order serial correlation of output is higher in the EMC model than in the standard RBC model while both consumption and investment display a weaker persistence. The comovement of aggregate consumption and investment with output are also weaker. If investment remains strongly correlated with output, the correlation of consumption with output falls strongly.

Table 2. Real Business Cycles and Endogenous Monitoring Costs compared

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This paradox of a more volatile output with less volatile consumption and investment is explained by the role the financial sector plays in the EMC model. The aggregate output of the EMC model includes a relatively low volatility sector one output while the volatility of sector two output is almost four times higher. Both of these figures are explained by the financial constraint. The response of sector one output to technological change is indeed hindered by the need for external finance while the activity of sector two is strongly procyclical. Since the size of the auditing sector is small, aggregate output is less volatile in the EMC model than in the standard RBC model. The financial constraint is also responsible for the higher volatility of aggregate consumption and investment. The risk neutral entrepreneurs strongly react to the technological changes in order to adapt their level of net wealth to an ever changing situation by quickly revising their saving-consumption decision. The high volatility of aggregate consumption and investment can thus be blamed on the very high volatility of entrepreneurial consumption. The higher persistence of aggregate output can also be accounted for by the financial constraint. The production of the consumption good is indeed constrained by the level of net wealth that entrepreneurs inherit from the past. Sector one output accordingly displays an higher level of persistence while sector two output responds without delay to each technological innovation and thus exhibits no persistence. The behavior of entrepreneurs is also responsible for the
weaker correlation of aggregate consumption and investment with output as illustrates the correlation of entrepreneurial consumption with aggregate output. The risk premium exhibits a negative first-order autocorrelation and a weak yet significantly positive correlation with output. The first of these properties is readily explained by the fact that a technological shock raises on impact the risk premium and the saving of entrepreneurs. In the following period, the net wealth of entrepreneurs has been modified and the risk premium falls dramatically. That the risk premium is procyclical stems from the fact that more consumption goods are produced during a boom before the net wealth of entrepreneurs has been modified. The level of debt is thus strongly procyclical and entails an higher rate of failures. The risk premium must be increased to cover the monitoring costs.

### Table 3. Endogenous Monitoring Costs: the financial sector

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<tr>
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<th>Standard deviation</th>
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<th>First-order autocorrelation</th>
<th>Contemporaneous correlation with output</th>
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</table>

When compared to the standard RBC model, the EMC model exhibits a hump-shaped response of output to a productivity shock and an higher persistence of aggregate output together with weaker co-movements of aggregate output with its components. Such properties are empirically relevant and the introduction of EMC can be seen to enhance the ability to describe stylized facts. Yet, a procyclical risk premium and an increase of failures during booms are quite counter factual.

### 5 Conclusion

We have embedded the Costly State Verification framework of Townsend (1979) in a general equilibrium model so as to obtain Endogenous Monitoring Costs. Previous models of this sort simply used the hypothesis of a deadweight cost which seems more suitable for a partial equilibrium approach. We have made the hypothesis that the cost of monitoring stems from the need to produce information, a good obtained, as any other, by the combination of capital and labor. We have also assumed that it is the quantity of information that comes as a proportion of the size of the project to audit. This has allowed us to study the effect on the economy of an innovation in the auditing
Our model delivers qualitative properties of some interest. The response of output to a positive productivity shock is hump-shaped. A technological innovation in the financial sector has a positive impact on aggregate output. This result is of particular interest since the standard hypothesis of proportional monitoring costs would lead to financial shocks having no effect whatsoever.

Quantitatively, our model is able to account for an higher persistence of aggregate output and reduces the co-movements of output with its components. Moreover, since we have computed the quantitative properties with the assumption of a technological shock hitting both sectors of our economy, our model is readily compared with the standard Real Business Cycles model.

In spite of these results, the introduction of a financial accelerator in a Real Business Cycles framework is disappointing. The Endogenous Monitoring Costs model displays counter intuitive properties concerning the behavior of failures and risk premium and does not notably differ from the standard Real Business Cycles model. This stems from the behavior of net wealth, for the most part inherited from the past. While net wealth responds on impact to a productivity shock via the rental rate of capital, this immediate increase is very weak and the input bill responds in the same way. The severe financial constraint imposed on the model thus fails to deliver a potent financial accelerator by lack of a strongly procyclical net wealth.

The potency of the financial accelerator then lies on the existence of various other frictions as in Bernanke, Gertler, and Gilchrist (1998) where convex costs of adjusting the capital stock and staggered price setting enhance the procyclicality of net wealth. A rather disturbing feature shared by this model and Bernanke, Gertler, and Gilchrist (1998) is the restriction of the financial relationship to the short term. Indeed, Thomas and Worrall (1990) show that the agency problem disappears when the financial relationship is enduring and the discount factor close to one. Long term financial relationships with a finite horizon must then be explored in order to establish whether the financial accelerator can be potent without resting on various other frictions.
References


Appendix A. The costly state verification framework

The optimal contract can be defined entirely by the size of the investment (i.e. the input bill, \(m_{it}\)) and the critical level of the idiosyncratic shock (\(\omega_{it}\)).

Using (4) and (6) to substitute away \(b_{it}\) in (7) and in (9) we can then write the entrepreneur’s and the bank’s expected profit as:

\[
\Pi_{it} = \theta_{i} F (k_{it}, h_{it}) \left( \int_{\omega_{it}}^{\infty} \omega_{it} d\Phi (\omega_{it}) - (1 - \Phi (\omega_{it})) \omega_{it} \right) \quad (A-1)
\]

\[
Z_{it} = \theta_{i} F (k_{it}, h_{it}) \left( (1 - \Phi (\omega_{it})) \omega_{it} + \int_{0}^{\omega_{it}} \omega_{it} d\Phi (\omega_{it}) - \frac{1}{p_{it}} \right) - \Phi (\omega_{it}) \nu_{it} y_{it} + n_{it} \quad (A-2)
\]

Since the production technology exhibits constant returns to scale, (21) and (22) can be written as

\[r_{t} k_{it}^{y} = \alpha \frac{y_{it}}{p_{it}} \quad \text{and} \quad w_{t} h_{it}^{y} = (1 - \alpha) \frac{w_{it}}{p_{it}}\]

which together with (2) yield: \(p_{it} m_{it} = \theta_{i} F (k_{it}, h_{it})\). (In the same way, one shows that \(\mu m_{it} p_{it} = y_{it}^{b}\))

We can then eliminate \(\theta_{i} F (k_{it}, h_{it})\) in A-1 and in A-2 and define the functions \(g_{1}\) and \(g_{2}\) as:

\[g_{1} (\omega_{it}) = \int_{\omega_{it}}^{\infty} \omega_{it} d\Phi (\omega_{it}) - (1 - \Phi (\omega_{it})) \omega_{it} \quad (A-3)\]

\[g_{2} (\omega_{it}) = (1 - \Phi (\omega_{it})) \omega_{it} + \int_{0}^{\omega_{it}} \omega_{it} d\Phi (\omega_{it}) - \Phi (\omega_{it}) \mu \nu_{t} \quad (A-4)\]

This allows us to write problem (10) in the text. Let \(\$\) be the Lagrangean of this problem and write the first order conditions:

\[
\frac{\partial \$}{\partial \lambda} = 0 \iff (1 - p_{it} g_{2} (\omega_{it})) m_{it} = n_{it} \quad (A-5)
\]

\[
\frac{\partial \$}{\partial m_{it}} = 0 \iff p_{it} g_{1} (\omega_{it}) = \lambda (1 - p_{it} g_{2} (\omega_{it})) \quad (A-6)
\]

\[
\frac{\partial \$}{\partial \omega_{it}} = 0 \iff \frac{\partial g_{1} (\omega_{it})}{\partial \omega_{it}} = -\lambda \frac{\partial g_{2} (\omega_{it})}{\partial \omega_{it}} \quad (A-7)
\]

Recall that \(\int_{0}^{\infty} \omega_{it} d\Phi (\omega_{it}) = 1\). Adding A-3 and A-4, then yields \(g_{1} (\omega_{it}) + g_{2} (\omega_{it}) = 1 - \Phi (\omega_{it}) \mu \nu_{t}\) which can be used to obtain (11) and (12) from the above conditions. We thus have a complete yet implicit definition of the optimal contract.
Appendix B. A general equilibrium model

- **Notations**
  - $c_t$: household consumption
  - $k_t^h$: household capital
  - $h_t$: household labor
  - $l_t$: household leisure
  - $c_t^e$: entrepreneurial consumption
  - $k_t^e$: entrepreneurial capital
  - $n_t$: net worth
  - $m_t$: input bill
  - $b_t$: debt
  - $\omega_t$: critical shock
  - $k_t$: total capital
  - $i_t$: total investment
  - $c_t^a$: aggregate consumption
  - $y_t^a$: aggregate output
  - $y_t^b$: information (sector 2 output)
  - $\theta_t^b$: productivity shock of sector 2
  - $k_t^b$: capital sector 2
  - $h_t^b$: labor sector 2
  - $r_t$: capital rental rate
  - $w_t$: wage
  - $\nu_t$: price of information
  - $r_t^b$: risk premium
  - $p_t$: mark-up
  - $\theta_t^r$: productivity shock of sector 1
  - $\phi_t^r$: price of information
  - $\mu$: mark-up
  - $\delta$: depreciation rate

- **Functional forms**
  - $u(c_t, l_t) = \ln(c_t) + vl_t$
  - $F(k_t^y, h_t^y) = (k_t^y)^\alpha (h_t^y)^{1-\alpha}$
  - $F^b(k_t^b, h_t^b) = (k_t^b)^a (h_t^b)^{1-a}$
  - $\phi(\omega) = \frac{1}{\alpha \omega_c^2 \pi^2} e^{-\frac{(\ln(\omega)-\mu)^2}{2\sigma^2}}$
  - $\Phi(\omega) = \int_0^\omega \phi(\omega) d\omega$

- **Households**
  - $v = \frac{1}{c_t} w_t$
  - $\frac{1}{c_t} = E \left[ \beta \frac{1}{c_{t+1}} (1 - \delta + r_{t+1}) \right]$

- **Firms**
  - $k_{t+1}^e + c_t^e = (1 - \delta + r_t) k_t^e + y_t \left( 1 - \mu \nu_t \int_0^{\omega_t} \phi_t (\omega) d\omega - \frac{1}{p_t} \right)$
  - $y_t = \theta_t^r (k_t^y)^\alpha (h_t^y)^{1-\alpha}$
  - $\theta_{t+1} = \theta_t^r e^{\mu \nu_t}$
  - $r_t = \alpha \frac{1}{p_t} \frac{y_t}{k_t^y}$
  - $w_t = (1 - \alpha) \frac{1}{p_t} \frac{y_t}{h_t^y}$
  - $n_t = (1 - \delta + r_t) k_t^e$
  - $m_t = \frac{n_t}{1 - \phi_2(\omega_t)}$
  - $b_t = m_t - n_t$
  - $\frac{1}{p_t} = 1 - \mu \int_0^{\omega_t} \phi_t (\omega) d\omega + \frac{g_1(\omega_t)}{\partial g_1(\omega_t) / \partial \omega_t}$
  - $1 = E \left[ \beta \xi (1 - \delta + r_{t+1}) p_{t+1} \frac{g_1(\omega_{t+1})}{1 - p_{t+1} g_1(\omega_{t+1})} \right]$. 

19
Banks

\[ y_t^b = \theta_t^b \left( k_t^b \right)^a \left( h_t^b \right)^{1-a} \]  
(B-13)

\[ \theta_{t+1}^b = \theta_t^b e^{\gamma_{t+1}} \]  
(B-14)

\[ r_t = \frac{\nu_t y_t^b}{p_t k_t^b} \]  
(B-15)

\[ w_t = \left( 1 - a \right) \frac{\nu_t y_t^b}{p_t h_t^b} \]  
(B-16)

\[ r_t^b = \frac{\omega_t y_t}{m_t - n_t} - 1 \]  
(B-17)

Market equilibria

\[ k_t^e + k_t^h = k_t^y + k_t^b \]  
(B-18)

\[ h_t = h_t^y + h_t^b \]  
(B-19)

\[ y_t^b = \mu \int_0^{\omega_t} \phi(\omega) \, d\omega y_t \]  
(B-20)

\[ y_t = c_t^a + i_t \]  
(B-21)

Miscellaneous

\[ k_t = k_t^e + k_t^h \]  
(B-22)

\[ l_t = 1 - h_t \]  
(B-23)

\[ i_t = k_{t+1} + (1 - \delta)k_t \]  
(B-24)

\[ y_t^a = y_t + \nu_t y_t^b \]  
(B-25)

\[ c_t^a = c_t + c_t^e \]  
(B-26)

\[ y_t = p_t m_t \]  
(B-27)
Appendix C. Solution

- Steady state

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For all calibrations : $\bar{v} = 1$ (since $\alpha = \alpha^b$), $\bar{\tau} = .0301$, $\sigma = .35529$, $ectyeps = .36681$

- Linear approximation around the steady state

The solution obtained is written as:

$S_{t+1} = MS_t + \epsilon_{t+1}$

$Y_t = \Pi Y_t$

$\hat{Y}_t = (\hat{y}^a_t \hat{y}^b_t \hat{\omega}_t \hat{\Phi}(\hat{\omega}_t) \hat{k}^y_t \hat{h}^y_t \hat{p}_t \hat{w}_t \hat{r}_t \nu_t \hat{h}_t \hat{l}_t)'$

$\hat{S}_t = (\hat{k}^e_t \hat{k}^h_t \hat{p}_t \hat{w}_t \hat{r}_t \nu_t \hat{h}_t \hat{l}_t)'$

$\hat{d}_t = (\hat{c}_t \hat{\epsilon}_t)'$

$M = \begin{pmatrix}
.9478 & -.0093 & .1640 & -.0016 \\
-.05020 & .0005 & .2589 & -.4791 \\
0 & 0 & .95 & 0 \\
0 & 0 & 0 & .95
\end{pmatrix}$

$\Pi = \begin{pmatrix}
.5369 & -.0053 & .4256 & .0112 \\
.1704 & 4.462 & -8.781 & 1.627
\end{pmatrix}$
\[
\Pi_x = \begin{pmatrix}
.0349 & .1507 & 1.633 & .0908 \\
.0322 & .1872 & 1.555 & .0974 \\
.1709 & -1.672 & 4.538 & .7588 \\
-1.611 & -.4670 & 8.20 & -.0820 \\
.5057 & .3757 & -.3597 & .1491 \\
1.029 & -.0289 & 0 & 0 \\
-.0289 & 1.007 & .0422 & .0046 \\
.0263 & .2667 & 1.337 & .1597 \\
.0305 & -.4092 & .6563 & .1455 \\
.1387 & -1.860 & 2.982 & .6614 \\
.9973 & .0365 & -.0781 & .0066 \\
-.5107 & .2720 & .9117 & .1484 \\
1.136 & -1.823 & 3.904 & -.3320 \\
-.3720 & -1.588 & 4.894 & -.1902 \\
.0059 & -.0795 & .2180 & -.0623 \\
.5369 & -.0053 & .4256 & .0112 \\
-.9710 & .2302 & 1.415 & .1530 \\
.1386 & -1.858 & 3.618 & .0222 \\
0 & 0 & 1 & -1 \\
-.5079 & .2355 & .9898 & .1418 \\
.2177 & -.1009 & -.4242 & -.0608
\end{pmatrix}
\]
Appendix D. Impulse response functions

- A productivity shock on the consumption good technology

Standard RBC model (---), Carlstrom and Fuerst (1998) (---), Endogenous monitoring (—)

Aggregate output

Capital

Investment

Consumption

Household consumption

Entrepreneurial consumption

Entrepreneurial wealth

Mark-up
- A productivity shock on the information technology
Risk premium

Failure probability

Output sector 1

Output sector 2

Labour (sector 1)

Capital (sector 1)

Labour (sector 2)

Capital (sector 2)