Is Growth Useful in RBC Models?

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February 2001

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Abstract: Standard RBC models fail to account for the business cycle stylized facts when growth results from a random walk in technology. Does this mean that we are to dismiss the evidence in favor of a unit root in output when we have a prior in favor of RBC models? To answer this question, we explore the usefulness of introducing an endogenous source of growth in a standard RBC model. Using a formal measure of fit, we show that doing so permits both to reproduce some key business cycle facts and to obtain a unit root in output.

Keywords: Endogenous growth, random walk, business cycle, real business cycle.

JEL classification: E32, O41

"Modern business cycle theory starts with the view that growth and fluctuations are not distinct phenomena to be studied with separate data and different analytical tools."

T.F. Cooley and E.C. Prescott (1995, p.4)

"If a model (...) produces a random walk in GNP, the results in this paper suggest that the parameters of that model should be picked to also generate interesting short-run dynamics of GNP, so that the variance of yearly changes in GNP is much larger than the variance of shocks to its random walk component."

J. H. Cochrane (1988, p. 897)

1 Introduction

Hansen (1997) and Ireland (2001) have recently shown that if Real Business Cycle (RBC) models are to give a correct picture of the business cycle, they need to incorporate very persistent yet stationary shocks. Consequently, if we are to follow the methodological guidelines suggested by Cooley and Prescott (1995), this unfortunate conclusion leaves no other choice but to model growth as a deterministic trend. Does this mean that we are compelled to dismiss the evidence of a unit root in output[1] when we have a prior in favor of RBC models?

To answer this question, this paper explores the consequences of introducing an endogenous source of growth in an otherwise standard RBC model. Our objective is to see to what extent this strategy enables us to eschew the apparent incompatibility between RBC models and the presence of a random walk in output. More specifically, we want to see if, following Cochrane's (1988) recommendations, it permits to obtain a realistic random walk in output and yet to reproduce key business cycle stylized facts in a satisfactory way.

The endogenous growth model which is considered is a slight modification of the model proposed by Jones et al. (2000).Two modifications are taken into account. First, we modify the human capital accumulation technology and use a functional form originally proposed by Lucas and Prescott (1971). Second, the model incorporates two productivity shocks, one affecting the production of final good and the other affecting the production of human capital.

[1] See the survey by Murray and Nelson (2000).

We confront the model with versions of the standard RBC framework with either deterministic or stochastic growth. We compare their ability at reproducing the business cycles stylized facts. To do so, we first resort to the usual RBC methodology consisting in simulating our models and computing summary statistics of the business cycle. Second, we use the test proposed by Watson (1993). This test amounts to finding how much error it is necessary to inject into a model so as to reproduce the spectral density of the data.

Our main results are as follows. Using Watson's formal measure of fit, we show that introducing a simple human-capital-based source of endogenous growth in an otherwise standard RBC model permits to reproduce key business cycle facts and yet to obtain a realistic random walk in output, thus offering an interesting alternative to either exogenous stochastic growth or exogenous deterministic growth. In particular, following Cochrane's (1988) recommendations, the special shape taken by our knowledge accumulation and the combination of our shocks permits to disentangle the reproduction of a random walk in output from the reproduction of key business cycle facts. In this sense, allowing for an endogenous source of growth allows us to eschew Hansen's (1997) dilemma. In particular we show that the model outperforms an RBC model featuring both stationary and permanent productivity shocks calibrated to reproduce both the variance and the spectrum at frequency zero of output growth. A side result is that endogenous growth correctly models growth when evaluated with Watson's (1993) test at very low frequencies.

Following the pioneering contributions by King and Rebelo (1986), many authors have successfully incorporated endogenous sources of growth in dynamic stochastic general equilibrium models and shown that this line of research can lead to substantial improvements over usual RBC models. For example, Einarson and Marquis (1997), Maffezzoli (2000) and Ozlu (1996) have shown that introducing an endogenous source of growth in RBC models helps reproduce some key comovements unaccounted for by standard RBC models. In this kind of environment, endogenous growth is not important per se but simply provides additional reallocation margins implying richer dynamics.

In contrast, authors such as Collard (1999) and Jones et al. (2000) have specifically studied how this line of research may help us understand the characteristics of cyclical growth. This paper is most closely linked to that of Jones et al. (2000). They simulate endogenous growth models and conclude that these are interesting for at least two reasons. First, they have a natural tendency to generate a high labor volatility. Second, these models are able to generate positive autocorrelation of output growth, a fact that RBC models cannot reproduce unless some frictions are added to the standard environment.

Nevertheless, our approach differs from theirs in three important dimensions. First, Jones et al. calibrate their models at annual frequency. As is common in the RBC literature, we rather choose to confront our model to quarterly data. Second, our resorting to Watson's (1993) test permits us to explicitly quantify how well our models account for the business cycle stylized facts. Third, we pay particular attention to the ability of the three models considered at reproducing both the variance of output growth and the size of the random walk in output, as defined by Cochrane (1988)[2]. In contrast, Jones et al. (2000) are not particularly interested in the long-run properties of their model.

Other authors, such as Dejong et al. (1996) or Perli and Sakellaris (1998) have shown that incorporating human capital in an RBC model might result in a dramatic improvement upon the performances of standard models. In particular, this accumulation margin provides additional propagation mechanisms which help generate a spectral density of output growth exhibiting a hump at business cycle frequencies.

The focus and objective of our paper differ from those of Dejong et al. (1996) and Perli and Sakellaris (1998). First, these authors do not allow for either endogenous growth or stochastic exogenous growth. Much as the bulk of the literature, they simply ignore the presence of a random walk in output. In contrast, we primarily focus on this particular point. Second, our goal is far more modest than their: the endogenous growth model which is proposed here is not designed to reproduce the shape of the spectral density of output growth. However, the model is able to generate a random walk in output as large as its empirical counterpart, yet doing as well as the best specification in Hansen's (1997) work.

The remainder is as follows. Section 2 describes the three models which we seek to compare. Each one is calibrated using as much a priori information as possible. Section 3 gives our result. To ease comparison with former RBC studies, we first apply the usual RBC methodology, which consists in simulating the models and computing summary statistics of the business cycle. We then briefly describe the general philosophy underlying Watson's (1993) test and then go on to apply it to the three models. Some sensitivity analyses are performed. The last section briefly concludes.

[2] Cochrane (1988) defines the size of the random walk in output as the variance of an innovation to the random walk component of output divided by the variance of output growth.

2 The Growth Models

In this section, we describe the three models which are considered. It is assumed that the reader has a good knowledge of the standard RBC model, so that we do not spend too much time on it. For further details, we refer the reader to the recent survey by King and Rebelo (1998).

2.1 Endogenous Growth

The economy is inhabited by a continuum of identical, infinitely-lived consumer endowed with one unit of time. Agents own all the primary factors, namely physical capital k_{t-1}, raw labor, n_t, and human capital, h_{t-1}. They combine physical capital and effective labor to produce a homogeneous final good which can be consumed or invested in the accumulation of either human or physical capital. Effective labor is defined as the product of raw labor and human capital, $n_t h_{t-1}$. We abstract from population growth. The representative agent's goal in life is to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \eta n_t\right] \tag{1}$$

subject to the constraints:

$$c_t + x_{kt} + x_{ht} = \zeta_t A k_{t-1}^{\phi} \left(n_t h_{t-1}\right)^{1-\phi}, \; A > 0, \;\; 0 < \phi < 1 \tag{2}$$

$$k_t = (1-\delta) k_{t-1} + x_{kt}, \; 0 < \delta < 1 \tag{3}$$

$$h_t = \gamma_t B h_{t-1}^{1-\theta} x_{ht}^{\theta}, \; B > 0, \;\; 0 < \theta < 1 \tag{4}$$

$$\log\left(\zeta_{t+1}\right) = \rho_\zeta \log(\zeta_t) + \varepsilon_{\zeta t+1}, \; 0 < \rho_\zeta < 1, \;\; \varepsilon_{\zeta t} \sim iid\,(0, \sigma_\zeta) \tag{5}$$

$$\log\left(\gamma_{t+1}\right) = \rho_\gamma \log(\gamma_t) + \varepsilon_{\gamma t+1}, \; 0 < \rho_\gamma < 1, \;\; \varepsilon_{\gamma t} \sim iid\,(0, \sigma_\gamma) \tag{6}$$

given an initial condition $(k_{-1}, h_{-1}, \zeta_0, \gamma_0)$ and subject to the usual positivity constraints. The term E_0 denotes the expectation operator, conditioned on information available as of time $t = 0$. β is the subjective discount factor. Equation (2) describes the resources constraint faced by the representative agent. The LHS lists the three alternative uses of final good: consumption, c_t, investment in physical capital, x_{kt}, and investment in human capital, x_{ht}. The RHS describes the Cobb-Douglas technology with which output is produced. A is a scale factor, ϕ is the share of physical capital, and ζ_t is a stationary productivity shock.

Equation (3) is the physical capital law of motion, where it is assumed that physical capital linearly depreciates every period with a constant rate δ. Equation (4) is the human capital law of motion. B is a scale factor, θ is the elasticity of next period's human capital with respect to x_{ht} and γ_t is another stationary shock. Such authors as Cassou and Lansing (1998), Collard (1999), Hercowitz and Sampson (1991) and Kocherlakota and Yi (1997) have resorted to similar functional forms based on that proposed by Lucas and Prescott (1971). It can be seen as reflecting adjustment costs in the accumulation of human capital. Notice that, whereas the above-mentioned authors used this hypothesis to obtain closed-form solutions in fully log-linear models, we do not seek to obtain such solutions. Instead, we use this hypothesis because it helps reproduce the size of the random walk in output, as defined by Cochrane (1988). Finally, equations (5) and (6) describe the assumed laws of motion for the productivity shocks ζ_t and γ_t.

Notice that we have imposed the indivisible labor hypothesis in (1). We do so because our main goal is to investigate to what extent allowing for an endogenous source of growth in a RBC model helps us to eschew Hansen's (1997) dilemma. Changing the specification of preferences would obscure our results.

2.2 Standard RBC Models

The endogenous growth model is compared with a standard RBC model with either deterministic or stochastic growth. In these models, the representative agent seeks to maximize (1) subject to the constraints (3) and:

$$c_t + x_{kt} = y_t \tag{7}$$

where y_t is the production of final good as of time t. In the deterministic growth model, we simply suppose that:

$$y_t = \zeta_t A k_{t-1}^{\phi} \left(n_t g^t \right)^{1-\phi} \tag{8}$$

where $g > 1$ is the constant (raw) growth rate and ζ_t is the same productivity shock as above in (5).

In the stochastic growth model, we suppose instead that:

$$y_t = Ak_{t-1}^{\phi} (n_t \xi_t)^{1-\phi} \tag{9}$$

$$\log \left(\xi_{t+1} \right) = \log (g) + \log (\xi_t) + \varepsilon_{\xi t+1}, \ \varepsilon_{\xi t} \sim iid (0, \sigma_\xi) \tag{10}$$

where $\log(g)$ is the drift, with g the same as above. In the calibration step, we will impose the same value for g in all three models.

2.3 Calibration

Before calibrating the model, we need to express the dynamic system summarizing the evolution of our economy in a stationary form. As is well known, due to the presence of endogenous growth, the model has a unit root and implies that all trending variables are cointegrated. Thus, we can pick any of them and divide the others by it. It is natural in this setup to select h_t. The resulting system is then log-linearized in the neighborhood of the associated deterministic steady state and solved using the undetermined coefficients method proposed by Uhlig (1999)[3]. It is easy from this solution to reconstitute the dynamic behavior of the log first difference of any of the original variables.

We now turn to the calibration of the structural parameters of our model. As in Jones et al. (2000), we face a tricky measurement problem: How should we count the expenditure flow x_h? Obviously, given our broad interpretation of human capital, a fraction of x_h should be counted as consumption expenditures (e.g. health care expenditures), and part of the remaining fraction as investment expenditures (e.g. R&D investment). More troublesome is the fact that some investment expenditures as on-the-job training or increases in the stock of organizational capital, clearly part of x_{ht}, are not counted in the National Accounts. Thus, it is difficult to tell what part of x_h should be counted as investment or as consumption. Unfortunately, we have no direct information to settle this question. Here, we take the extreme of counting all x_h as consumption. We thus define total measured consumption c_{mt} in period t as: $c_{mt} = c_t + x_{ht}$. None of the results reported below are qualitatively changed if we choose either to count all x_h as measured investment. In the remainder, when we refer to c or consumption, it is intended that the concept should be interpreted as c_m.

[3] A technical appendix is available from the author upon request.

The average output growth over the period 1965:1-1995:4[4] is 0.43%; we consequently set $g = 1.0043$. We set the steady state interest rate to 1%, so that $\beta = 0.994$. We follow Hansen (1997) and set labor's share in output to 64%, thus $\phi = 0.36$. We set $\delta = 0.01$ so that the consumption share is close to its empirical counterpart (near 74%). The parameter η is pinned down so that the representative agent devotes 30% of his time endowment to the market, thus $n = 0.3$ and $\eta = 3.214$.

When it comes to the elasticity of h with respect to x_h, we face a double constraint. First, this parameter determines the steady state ratio x_h/y which, according to Cassou and Lansing (1999) should be near 8%. Second, this parameter determines the long-run effect of a shock to γ_t or ζ_t, hence governs the size of the random walk in y_t, as defined by Cochrane (1988). We set the steady state ratio x_h/y to 8.3% by imposing $\xi = 0.0008$. Thus, this value delivers approximately the same value for x_h/y as that retained by Cassou and Lansing (1999) and, as described below, permits to reproduce the size of the random walk in output with a small variance for γ_t. We then impose $B = 1.006$ so as to meet our requirement on g, given the steady state value of x_h/h and θ. Finally, we impose that the steady state value of y/h is normalized to unity. We thus set $A = 0.763$. The calibration is summarized in table 1.

It remains for us to calibrate the stochastic processes parameters ρ_ζ, ρ_γ, σ_ζ and σ_γ. We follow Hansen's (1997) recommendations and set $\rho_\zeta = 0.9$. Following DeJong et al. (1996), we (somewhat arbitrarily) suppose that γ_t is persistent and set $\rho_\gamma = 0.9$. We then choose σ_ζ and σ_γ so that the model reproduces both the variance and the frequency zero spectral density of output growth. The spectral density of output growth is obtained from a cointegrated VAR with variables $(\Delta\widehat{y}_t, \widehat{c}_t - \widehat{y}_t, \widehat{x}_t - \widehat{y}_t, \widehat{n}_t)$ with three lags, where a letter with a hat denotes the natural logarithm of the associated variable and the sign Δ stands for the first difference filter. The lag in the VAR is chosen by minimization of the Schwartz information criterion. The exact values are: $\sigma_\zeta = 0.4502\%$ and $\sigma_\gamma = 0.0491\%$. Notice that our procedure compels the model to exactly reproduce the relative size of the random walk in output, as defined by Cochrane (1988). To complement our analysis, we will also study to a specification of the endogenous growth model without shocks to γ_t. In this case, we cannot simultaneously reproduce the variance of output growth and its spectrum at frequency zero. We then select σ_ζ so as to simply reproduce the variance of $\Delta\widehat{y}_t$. The exact choice is $\sigma_\zeta = 0.4687\%$. Notice that it is possible to reproduce the spectrum

[4] The data are described in appendix A.

at frequency zero of output growth, but at the cost of imposing unreasonably high a variance for ζ_t.

We now give the details pertaining to the two RBC models. We still impose $g = 1.0043$ and $R = 1.01$. We normalize the steady state value of output in both cases. More precisely, we set the steady state value of y_t/g^t in the deterministic growth model or y_t/ξ_t in the stochastic growth model to one. In both cases, we set $n = 0.3$. The parameters ϕ, β and δ take the same values as in the endogenous growth model. We then set $\rho_\zeta = 0.9$, and select σ_ζ and σ_ξ so that the exogenous growth models exactly reproduce the variance of output growth[5]. The chosen values are $\sigma_\zeta = 0.4671\%$ and $\sigma_\xi = 0.9166\%$.

3 Simulations

We now turn to our results. As mentioned in the introduction, we start by comparing the three models using the usual RBC methodology, i.e. we simulate each model and apply the Hodrick-Prescott filter to the artificial time-series. Then, we take the model to the data using Watson's (1993) test. This step is designed to quantify and formally confirm the preceding results.

3.1 Usual RBC Methodology

To ease comparison with former RBC studies, both with or without endogenous growth, we now propose to simulate each of our models and compare the results with our sample ranging from 1965:1 to 1995:4. The cyclical component of each series is defined as the residual after extraction of a smooth trend using the Hodrick-Prescott filter. Since we are working with quarterly time series, we set the smoothing parameter to 1600. For each model, we draw 100 time series of 124 periods for output, consumption, investment, hours and average labor productivity. Standard deviations as well as correlations are computed, averaged over our 100 draws and compared with their empirical counterparts. The results are reported in table 2. Panel A reports the statistics computed for output, consumption, investment, hours and average labor productivity in our sample. Panel B reports similar statistics for the endogenous growth model with two shocks (ζ_t and γ_t). Panel C gives the results with the single-shock specification of the endogenous growth model. Panels D and E report results for the trend stationary and difference stationary RBC models,

[5] When it comes to the stochastic growth version of the basic RBC model, we could have alternatively selected σ_ξ so as to reproduce the spectral density of output growth at frequency zero. This strategy would have made the model look bad for the wrong reason: the spectrum of output growth at frequency zero is very small compared to the peak it reaches at business cycle frequencies.

respectively.

As is clear form the figures reported, none of our models can replicate the standard error of the HP cyclical component of output. This first result is interesting since it simply states that neither standard RBC models nor our endogenous growth model can replicate both the variance of output growth and the variance of the cyclical component of output. For this reason, the second column of table 2 may be more informative.

As emphasized by Hansen (1997), an RBC model with stochastic growth suffers from two major shortcomings which are confirmed in panel E. First, it cannot reproduce the relative volatility of labor. Second, it fails at replicating the relative volatility of investment. Moreover, it clearly predicts too high a correlation between hours and average labor productivity. On the first two shortcomings, a specification with deterministic growth clearly does a far better job. Unfortunately, it still predicts a high correlation between n and y/n. Notice that our results do not exactly confirm Hansen's (1997) in terms of the absolute volatility of output. To exactly reproduce his results, we would have to set the restriction $\sigma_\zeta = \sigma_\xi / (1 - \phi)$. In contrast, the variance of our shocks in the RBC models are selected to reproduce that of output growth. Thus, σ_ξ is much higher than σ_ζ. In other words, the RBC model with stochastic growth does a very poor job at reproducing the key business cycle stylized facts.

When it comes to our endogenous growth model, we obtain a substantial improvement over the difference stationary RBC model. Notice however that the endogenous growth model with only shocks to ζ_t is virtually indistinguishable from the RBC model with trend stationary technical progress. Comparing panels C and D in table 2, we do not see any quantitatively significant difference between the two models. Nevertheless, there exists one which the HP filter necessarily hides[6]: the endogenous growth model generates a random walk in output (and the other growing variables). Using the definition given by Cochrane (1988), however, we obtain a random walk the size of which is more than 20,000 times as small as its empirical counterpart. Thus, in this case, everything is as if we had only modelled the deterministic trend in technical progress: the random walk in output is so small that it could easily be ignored by a formal stationarity test. Adding a shock to γ_t substantially modifies this picture.

When we take two shocks into account, we obtain even more encouraging results. The relative

[6] King and Rebelo (1993) have shown that when we apply the HP filter on a time series, we implicitly first-difference four times this variable.

volatilities of investment and labor roughly match their empirical counterparts. Moreover, the correlation between average labor productivity and hours has extraordinarily decreased. In a somewhat different context, Ozlu (1996) obtained a similar result. It is clear that this important improvement results from the introduction of an additional shock, namely γ_t, in the model. Even though the relative volatility of consumption is twice as small as its empirical counterpart, it is substantially higher than in the trend stationary RBC model. Thus, globally, our endogenous growth model is at least as good as the RBC model with deterministic growth and clearly beats it when we take two shocks into account.

What makes the endogenous growth model look so good? It can be shown that a shock to ζ_t explains the major part of the variance of output growth in this model. On the contrary, a shock to γ_t essentially explains the random walk in output, but does not contribute much to the variance of $\Delta \hat{y}_t$. In other words, the instantaneous response of output to a shock to a one-standard-error innovation in ζ_t is much higher than the associated long-run effect, which makes the endogenous growth model resemble a trend stationary RBC model. On the contrary, the long-run effect of a one-standard-error shock to γ_t is higher than the associated instantaneous response of output, which is very small. Thus, if there were only shocks to γ_t, the model would resemble a difference stationary RBC model. With both shocks to ζ_t and to γ_t, we obtain what is best in each world. More precisely, following Cochrane's (1988) recommendations, the particular combination of our shocks thus permits to disentangle the reproduction of a random walk in output from the reproduction of the key business cycle facts: the HP-filtered movements of output generated by a shock to ζ_t are always bigger than those generated by a shock to γ_t. In this sense, allowing for an endogenous source of growth has allowed us to eschew Hansen's (1997) dilemma.

To sum up, the endogenous growth model is able to correctly reproduce the size of the random walk in $\hat{y}_t$ and, at the same time, can catch the key features of the business cycle. However, up to now, we have no idea as to the the quantitative importance of correctly reproducing the random walk in output. In the following exercises, the business cycle is defined as the residual after HP-filtering the main macroeconomic aggregates. Since HP-filtering eliminates the low-frequency movements in the variables, the methodology used in this section is of no help to answer this question. Consequently, based on the statistics reported in this section, it seems difficult to decide between the endogenous growth model and the trend stationary RBC model. To gain further information, we now turn to Watson's (1993) test.

3.2 Quantifying the Goodness of Fit

In this subsection, we describe the philosophy underlying Watson's (1993) test before implementing it so as to assess which growth hypothesis fits the data best.

3.2.1 Describing Watson's (1993) Test

Watson's (1993) test allows us to synthesize the ability of a model to reproduce both the business cycle stylized facts and the low frequency behavior of the data. Indeed, it consists in decomposing the performances of a model into the frequency domain. The method might be described as follows. Assuming that the empirical and simulated data are jointly stationary, we can define the error induced by the model as the difference between the two data sets. Let d_t and τ_t denote, respectively, the empirical and simulated data, and define naturally the approximation error as $u_t = d_t - \tau_t$. To fix the ideas, we will study the spectral properties of the process $d_t = (\Delta\hat{y}_t, \Delta\hat{c}_t, \Delta\hat{\imath}_t, \Delta\hat{n}_t)$. The method proposed by Watson (1993) consists in minimizing a weighted trace of the variance of the error u_t which we have to plug into the model τ_t so that the spectral density matrix of the latter corresponds to that of d_t. Given the definition of u_t, we can define the spectral density matrix of u at frequency ω by the formula:

$$F_u\left(e^{-i\omega}\right) = F_d\left(e^{-i\omega}\right) + F_\tau\left(e^{-i\omega}\right) - F_{\tau d}\left(e^{-i\omega}\right) - F_{d\tau}\left(e^{-i\omega}\right)$$

where $F_a\left(e^{-i\omega}\right)$ is the spectrum of a at frequency ω (if a is an n dimensional vector, $F_a(\cdot)$ is an $n \times n$ matrix), and $F_{ab}\left(e^{-i\omega}\right)$ is the cross-spectrum of a and b. We resort to the state space form of the approximate solution to the model to compute $F_\tau\left(e^{-i\omega}\right)$. To obtain $F_d\left(e^{-i\omega}\right)$, as proposed by King et al. (1991) and Watson (1993), we use the parameters from the cointegrated VAR with variables $(\Delta\hat{y}_t, \hat{c}_t - \hat{y}_t, \hat{\imath}_t - \hat{y}_t, \hat{n}_t)$ previously estimated. In contrast, the cross spectrum $F_{\tau d}\left(e^{-i\omega}\right)$ cannot be estimated, and is rather chosen so as to minimize a weighted trace of the variance of u_t subject to the constraint that the spectral density matrix of (τ_t, d_t) is positive semidefinite at all frequencies. In the following application, we will give equal weight to each of the components of u_t.

For each frequency, we can determine a lower bound of the variance of the approximation error divided by the variance of the data; let $r(\omega)$ denote this bound and $r_j(\omega)$ denote the jth component of r. In the same fashion, let $F_a\left(e^{-i\omega}\right)_{jj}$ denote the (j, j) element of matrix F_a. We can then define:

$$r_j(\omega) = \frac{F_u\left(e^{-i\omega}\right)_{jj}}{F_d\left(e^{-i\omega}\right)_{jj}}$$

Watson (1993) proposes to integrate separately both the numerator and denominator of the above expression, defining so the relative mean square approximation error (RMSAE) which the model induces compared with the data. The smaller it is, the better the model reproduces the spectral behavior of component j in the vector d. We can then choose to highlight the quality of the model by focusing on a particular frequency band.

3.2.2 Applying Watson's (1993) Test

We now proceed to implement the above-described methodology in all our three models. We start this exercise with the variables taken in log-first difference. The associated $r_j(\omega)$ are integrated over four different frequency ranges. First, we focus on the interval $[0, \pi]$, which permits us to see how well a given model globally behaves. Second, this interval is narrowed to $[0, \pi/3]$, so as to emphasize the second order properties of the models over business cycle and growth frequencies -i.e. periods ranging from 6 quarters to infinity. The last interval is then broken down into two separate ranges, one for the business cycle, $[\pi/16, \pi/3]$ (period 6 to 32 quarters), and one for the long-run, $[0, \pi/16]$ (32 quarters to infinity). Finally, we apply the HP-filter to the variables taken in level, and integrate the $r_j(\omega)$ over $[0, \pi]$, as proposed by Watson (1993). The results are reported in table 3.

Using a different approach than in the previous quantitative exercise, we obtain very similar results. In particular, comparing panel D with any of the panels A, B or C, it appears that a difference stationary RBC model always produce higher RMSAE than other models. Globally, this result strongly confirms Hansen's (1997) and Ireland's (2001) conclusions.

Comparing panels B and C, we see that a trend stationary RBC model slightly outperforms the endogenous growth model with only shocks to ζ_t. Recall that in this case, neither model is able to reproduce the random walk in output, i.e. the models necessarily behave poorly at very low frequencies (for example over the frequency range $[0, \pi/32]$). In contrast, augmenting the endogenous growth model with an additional shock devoted to reproducing the random walk in output, substantially improves our results.

Comparing panels A and C, we see that both the trend stationary RBC model and the two-shock endogenous growth model have similar performances when it comes to investment. However, the endogenous growth model seems better at reproducing the behavior of consumption or output. The opposite conclusion obtains with total hours over the frequency range $[0, \pi]$. However, restricting the frequency band to $[0, \pi/3]$, the two-shock endogenous growth model uncontroversially outperforms the trend stationary RBC model.

Last but not least, notice that the exogenous deterministic growth hypothesis outperforms the exogenous stochastic growth hypothesis over the frequency range $[0, \pi/16]$, which corresponds to medium-run to long-run movements. This result substantially reinforces the conclusion previously obtained by Hansen (1997) and Ireland (2001). They found that modelling growth as resulting from an exogenous stochastic trend in technical progress deteriorates the performance of stationary RBC models at business cycle frequencies. We found that over a lower frequency range, it is actually beaten by a model with no random walk at all! It is reassuring to see that in this dimension, our minimal endogenous growth hypothesis gives the best results: endogenous growth correctly models growth. As is clear from the comparison of panels A and B, this positive result stems from the introduction of γ_t. If this additional shock is omitted, the endogenous growth model still beats the difference stationary RBC model over the frequency range $[0, \pi/16]$, but the trend stationary RBC gives slightly better results.

Using Watson's (1993) test has permitted to globally confirm our previous conclusions. However, we now have a precise idea as to the relative importance of reproducing the size of the random in output. As mentioned above, when the endogenous growth model is only hit by shocks to ζ_t, it has "almost" no random walk in output and is very similar to the trend stationary RBC model. On the contrary, the two-shock specification, which exactly reproduces the size of the random walk, globally exhibits the smallest RMSAEs. However, the good performances of this specification do not just rely on the correct reproduction of the random walk in output. To see this, notice that it exhibits small RMSAEs over the frequency band $[\pi/16, \pi/3]$ which eliminates long-run movements. We suspect that this result is linked to the particular functional form retained for the accumulation of human capital.

3.3 Sensitivity Analysis

As mentioned before, the particular shape that is taken by our human capital accumulation has been chosen because it helps disentangle the reproduction of the variance of output growth from the reproduction of the its spectrum at frequency zero. Even though there is no "standard" functional form for this accumulation technology, one may wonder how sensitive are our results to the particular assumptions made in eq. (4). To help answer this question, we propose to replace this equation with the more conventional functional form:

$$h_t = (1-\theta)\, h_{t-1} + \gamma_t B x_{ht} \tag{11}$$

The parameter θ is then interpreted as the depreciation rate of human capital. We then face two difficulties with this specification. First, as explained in the calibration, x_h/y should be near 8%. However, when we impose $\theta = 0$, which is a reasonable value with the specification in (11), we obtain $x_h/y = 0.275$. Setting θ to a higher value results in an even bigger ratio x_h/y when we impose the general restrictions previously discussed. Second, this functional form cannot be used to reproduce the spectrum of output growth at frequency zero. Doing so would result in a HP-filtered consumption negatively correlated with HP-filtered output. Here, we simply ignore γ_t and set $\sigma_\zeta = 0.2907\%$ so as to simply reproduce the variance of output growth.

The results are reported on table 3 (panel F). Using Watson's (1993) test, we obtain similar results for output and significantly deteriorated results for investment and consumption compared with the trend stationary RBC model. Moreover, the random walk in output is about 20 times as small as its empirical counterpart. Notice however that our results confirm the conclusions of Jones et al. (2000), in that the model with a linear accumulation for human capital needs a relatively small RMSAE for reproducing the spectrum of total hours growth.

Thus, it derives from this sensitivity analysis that the particular shape taken by our human capital accumulation in (4) helps take Cochrane's (1988) recommendations into account. In combination with our structural productivity shocks, it helps reproduce the random walk in output without deteriorating the ability of the model to reproduce key features of the business cycle. In contrast, a more conventional specification like in eq. (11) does not have the same amount of flexibility.

If disentangling the reproduction of the random walk in output from the reproduction of the variance of output growth is so important, why not considering an RBC model with both stationary and permanent productivity shifts? Hansen (1997) proposed a first approach following this line of reasoning and considered a single productivity shock with a permanent and transitory components. Here, we propose to compare our two-shock endogenous growth model with a two-shock RBC model. Formally, we suppose that the representative agent seeks to maximize (1) subject to the constraints (7) and:

$$y_t = \zeta_t A k_{t-1}^{\phi} \left(n_t \xi_t\right)^{1-\phi} \tag{12}$$

where ζ_t and ξ_t evolve according to (5) and (10), respectively. We set $\rho_\zeta = 0.9$, $\sigma_\zeta = 0.4854\%$ and $\sigma_\xi = 0.4164\%$.

The results are reported on table 3 (panel E). Compared with our two-shock endogenous growth model, this specification needs a lower RMSAE to reproduce the spectrum of consumption growth over the frequency ranges $[0, \pi/3]$, $[\pi/16, \pi/3]$ and $[0, \pi/16]$. The model otherwise seems to obtain somewhat lower performances than the two-shock endogenous growth model. In particular, it needs a higher RMSAE for output and investment growth. It follows from these results that our endogenous growth model is better armed than a two-shock RBC model to disentangle the reproduction of the random walk in output from the variance of output growth.

4 Conclusion

In this paper, we have shown that the way in which one models growth might heavily impact on the performances of an RBC model, thus confirming the conclusions previously obtained by Hansen (1997) and Ireland (2001). Nevertheless, in contrast to them, we do not conclude that modelling growth as resulting from an exogenous deterministic trend is the one best way to mimic the second order properties of the relevant data.

Using either the usual RBC methodology or the test proposed by Watson (1993), we have shown that introducing an endogenous source of growth might prove far better an assumption. Indeed, introducing a simple human-capital-based source of endogenous growth in an otherwise standard RBC model permits to reproduce key business cycle facts as well as to obtain a realistic random walk in output, thus offering an interesting alternative to either exogenous stochastic growth or exogenous deterministic growth.

Moreover, our results suggest that it might be important to distinguish productivity shocks according to their contribution to the spectral decomposition of the variance of output growth. As in a trend stationary RBC model, standard productivity shocks have good properties in our model because they mainly affect the variance of output growth over business cycle frequencies. On the contrary, shocks impacting the human capital accumulation help get even better results because they mainly affect the variance of output growth at very low frequencies. Comparing this setup with an RBC model with both stationary and permanent productivity shifts, we obtained that our model represents a better approximation of real world fluctuations.

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Appendix

A Data Description

So as to define our macroeconomic variables, we start from the following time series:

[1] : consumption of durable goods;

[2] : consumption of non durable goods;

[3] : consumption of services

[4] : private fixed investment;

[5] : average weekly hours;

[6] : total non farm employment;

[7] : civilian population over 16;

The quarterly time series $[1]-[4]$ are taken from the NIPA, over the period 1964(4)-1995(4); the first point is used to define the growth rate of output. The monthly series $[5]-[7]$ are taken from the BLS over the period 1964(1)-1995(12); they are transformed to quarterly frequency by simply taking the average value over a quarter. We then define the aggregate series:

$c : ([2] + [3])/[7];$

$x : ([1] + [4])/[7];$

$y : c + x;$

$h : [6] * [5]/[7];$

Notice that public expenditures are excluded from our definition of c, x and y. Finally, average labor productivity is defined as the ratio y/n.

Table 1
Calibration Summary[a]

Panel A: Structural Parameters			
Parameter	Value		Interpretation
β	0.9943		Subjective discount factor
η	$\{3.2144, 2.8724\}$		Leisure weight in utility
A	0.7634		Scale parameter in the final good technology
ϕ	0.3600		Share of capital in the final good technology
δ	0.0100		Depreciation rate of physical capital
θ	0.0008		Elasticity of h wrt x_h
B	1.0063		Scale parameter in the knowledge technology
Panel B: Stochastic Shocks			
Parameter	Model	Value	Interpretation
ρ_γ	EGR 1	0.9000	Persistence of γ_t
ρ_ζ	EGR 1, EGR 2, RBC 1	0.9000	Persistence of ζ_t
σ_γ	EGR 1	0.0491	S.E. of ε_γ in %
σ_ξ	RBC 2	0.9166	S.E. of ε_ξ in %
σ_ζ	EGR 1	0.4502	S.E. of ε_ζ in %
—	EGR 2	0.4687	—
—	RBC 1	0.4671	—

[a]Calibration of the structural parameters. The first value of η in panel A is for the endogenous model and the second for the RBC models. Model codes in panel B: EGR 1, benchmark endogenous growth. EGR 2, endogenous growth model with only shocks to ζ. RBC 1, trend stationary RBC model, RBC 2, difference stationary RBC model.

Table 2
Standard Errors and correlations[a]

			Correlation Matrix				
Variable	Standard Error	S.E. Rel. to y	y	c	x	n	y/n
Panel A: Data							
y	1.83	1.00	1.00				
c	0.89	0.49	0.88	1.00			
x	6.65	3.64	0.94	0.74	1.00		
n	1.64	0.90	0.83	0.73	0.77	1.00	
y/n	1.01	0.55	0.45	0.39	0.45	-0.12	1.00
Panel B: EGR 1							
y	1.38	1.00	1.00				
c	0.29	0.21	0.33	1.00			
x	5.14	3.73	0.99	0.17	1.00		
n	1.31	0.95	0.98	0.12	0.99	1.00	
y/n	0.29	0.21	0.33	1.00	0.17	0.12	1.00
Panel C: EGR 2							
y	1.39	1.00	1.00				
c	0.19	0.14	0.76	1.00			
x	5.00	3.59	0.99	0.72	1.00		
n	1.25	0.90	0.99	0.70	0.99	1.00	
y/n	0.19	0.14	0.76	1.00	0.72	0.70	1.00
Panel D: RBC 1							
y	1.38	1.00	1.00				
c	0.19	0.14	0.76	1.00			
x	4.98	3.60	0.99	0.71	1.00		
n	1.25	0.90	0.99	0.69	0.99	1.00	
y/n	0.19	0.14	0.76	1.00	0.71	0.69	1.00
Panel E: RBC 2							
y	1.47	1.00	1.00				
c	0.72	0.49	0.99	1.00			
x	3.68	2.50	0.99	0.98	1.00		
n	0.77	0.52	0.99	0.96	0.99	1.00	
y/n	0.72	0.49	0.99	1.00	0.98	0.96	1.00

[a]Standard errors and correlation matrix for the quarterly sample 1965:1-1995:4 and for the models. The figures reported for the latter are sample means of statistics computed for each of 100 simulations of 124 periods. Both artificial and actual series are logged and HP-filtered, setting the smoothing parameter to 1600. The exogenous stochastic disturbances are calibrated as described in the text. The standard deviations have been multiplied by 100. Volatility relative to output is the standard deviation divided by the standard deviation of output. Model codes as in table 1.

Table 3
RMSAE[a]

Frequency range	Filter	Panel A: EGR 1				Panel B: EGR 2			
		$\hat{y}$	$\hat{c}$	$\hat{n}$	$\hat{\imath}$	$\hat{y}$	$\hat{c}$	$\hat{n}$	$\hat{\imath}$
$[0,\pi]$	Δ	0.1910	0.5542	0.0871	0.9988	0.2283	0.9677	0.0872	0.9395
$[0,\pi/3]$	Δ	0.2141	0.5831	0.1188	0.4319	0.2379	0.8202	0.1297	0.4360
$[\pi/16,\pi/3]$	Δ	0.2136	0.6075	0.1177	0.3852	0.2331	0.8384	0.1283	0.3861
$[0,\pi/16]$	Δ	0.2407	0.5181	0.1501	0.6201	0.2878	0.7558	0.1641	0.6386
$[0,\pi]$	HP	0.2802	0.6529	0.1719	0.4641	0.2978	0.8031	0.1859	0.4753

Frequency range	Filter	Panel C: RBC 1				Panel D: RBC 2			
		$\hat{y}$	$\hat{c}$	$\hat{n}$	$\hat{\imath}$	$\hat{y}$	$\hat{c}$	$\hat{n}$	$\hat{\imath}$
$[0,\pi]$	Δ	0.2283	0.9669	0.0872	0.9420	0.3089	2.6897	0.3085	2.2288
$[0,\pi/3]$	Δ	0.2378	0.8234	0.1288	0.4352	0.2739	1.2657	0.2097	0.5757
$[\pi/16,\pi/3]$	Δ	0.2330	0.8418	0.1275	0.3856	0.2720	1.4121	0.2060	0.5268
$[0,\pi/16]$	Δ	0.2874	0.7588	0.1630	0.6362	0.2902	0.6937	0.2652	0.7751
$[0,\pi]$	HP	0.2977	0.8069	0.1848	0.4741	0.2907	0.8844	0.2900	0.6911

Frequency range	Filter	Panel E: RBC 3				Panel F: EGR 3			
		$\hat{y}$	$\hat{c}$	$\hat{n}$	$\hat{\imath}$	$\hat{y}$	$\hat{c}$	$\hat{n}$	$\hat{\imath}$
$[0,\pi]$	Δ	0.2706	1.8875	0.0925	1.1448	0.1925	4.1775	0.4977	0.9007
$[0,\pi/3]$	Δ	0.2262	0.5146	0.1498	0.4662	0.2210	1.5138	0.5943	0.3921
$[\pi/16,\pi/3]$	Δ	0.2255	0.5381	0.1486	0.4166	0.2174	1.8450	0.5930	0.3556
$[0,\pi/16]$	Δ	0.2541	0.4454	0.1844	0.6682	0.2662	0.2977	0.6243	0.5423
$[0,\pi]$	HP	0.2982	0.6547	0.2088	0.5188	0.2869	0.9462	0.6392	0.4302

[a] In all the reported experiments, we give equal weight to the four variables. The sign Δ refers to the first-difference filter, and HP refers to the Hodrick-Prescott filter, with smoothing parameter set to 1600. Model codes as in table 1 and EGR 3: endogenous growth model with a linear accumulation of human capital, RBC 3: RBC model with both stationary and permanent productivity shocks