Business Cycles Asymmetry and Monetary Policy: A Further Investigation using MRSTAR Models

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Abstract

This paper investigates the asymmetric effects of monetary shocks when the impact of monetary policy on real activity works through state-dependent variables. We use a nonlinear model, the Multiple Regime Smooth Transition Autoregressive model (MRSTAR), that allows the effects of shocks to vary across the business cycles when monetary innovations modify both the endogenous and state variables. Our impulse response functions show a history dependence property. Indeed, hitting the economy at a given time induces persistence and asymmetric responses across histories and shocks. The empirical application concerns the U.S. over the period 1975:1 to 1998:2.

Key words: Asymmetry - Nonlinear models - Monetary impulses

JEL classification: C32, E37, E32, E52

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1 Introduction

The past years have witnessed an increasing number of papers dealing with the asymmetry of business cycles. Even though the idea is very old, empirical studies have grown exponentially since the beginning of the eighties. The recognition that structural changes affect the economies at any period has encouraged the use of multiple regimes models, instead of previous empirical approaches that mostly distinguished between two phases in the business cycles: expansion and depression. The idea according to which there exist variations in the slopes of expansion and depression phases that induce time variations from the mean to the trough or peak of cycles, is an old idea - at least it dates back to Mitchell (1927) -. However, for a long time, the problem has been the following: how can the theoretical concept be made operational? In an attempt to answer this question, econometricians have suggested the use of nonlinear time series models that enable the study of different dynamics over the business cycles. A plethora of papers on this topic started emerging in the eighties and in the nineties (among others Neftçi (1984), Falk (1986), Lüükkonen and Teräsvirta (1991), Anderson and Teräsvirta (1992), Emery and Koenig (1992), Sichel (1994), Ramsey and Rothman (1996), Verbrugge (1997), Pesaran and Potter (1997), van Dijk and Franses (1999)).

Among the arguments that motivate the use of nonlinear structures, a simple idea is that the output fluctuations are influenced by variables that distort the business cycle shape. Such variables cause changes in regime in the sense that output variations follow a different time series process over different periods. This may be a cause of asymmetric dynamics. The use of nonlinear models of the business cycle imposes to clarify the meaning of 'asymmetry'. With regard to linear or VAR models, the 'asymmetry' of the business cycles suggests that depressions last a longer period than expansions, or that shocks have stronger effects on certain variables during one of the two phases. With regard to nonlinear models, the meaning of 'asymmetry' is more general in the sense that we simply say that shocks have time-varying effects on the real activity. This variability occurs because the parameters of the equations describing the dynamics of the output change as a result of a regime-shift variable. Such a view severely distorts our comprehension of how demand and supply shocks contribute to movements in the real GNP over the business cycle. Indeed, when one perturbs the present to produce information on the dynamics of a nonlinear model, the response not only depends on the sign of the shocks, but is also a function of the history of the GNP and of the magnitude of the shocks. This is a new challenge to econometricians.

In this paper, we study the effects of monetary shocks on the real sec-
tor of the US economy, assuming that output fluctuations are governed by regime-shift models, here the Multiple Regime Smooth Transition Autoregressive models (henceforth MRSTAR). These were introduced by van Dijk and Franses (1999) who analyze how regime-shift variables cause asymmetries in the US business cycle. They generalize the STAR models that were extensively used in the literature. Why is it interesting to use an MRSTAR model to evaluate the asymmetric effects of monetary shocks on real GNP? If we were using a linear model (for instance a VAR process), we would proceed as follows. We would, firstly estimate a money-output equation, secondly create two series of respectively positive and negative monetary shocks, and thirdly study the properties of impulse response functions. In such a framework, the usual results obtained in the literature may be summarized as follows: (1) money does affect output strongly when monetary policy is restrictive and raises inflation when it is expansive; (2) the effects of money on output is greater during the depression phases of the business cycle and their impact on inflation are greater during expansion phases; (3) if prices adjust slowly, then only negative shocks affect the output. In a MRSTAR model, contractionary and expansionary monetary shocks lead asymmetric effects that differ significantly from those just mentioned. Indeed, the impulse response functions exhibit a time dependence property. The coefficients of the money-output equation are indeed state-dependent and vary according to transition variables that generate changes in the business cycle regimes. The regime-shift variables are economic indicators characterizing both the aggregate supply and the aggregate demand. For instance, the reaction of output to negative monetary shocks is undetermined. Real GNP can either increase or decrease. This indeterminacy occurs because the level of stocks and the production capacity act as state variables that condition the reaction of the GNP to money variations (see Wong (2000)). There are other state variables that induce time variation of the elasticity of output to money. Firstly, due

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to the imperfect structure of the credit market, initial shocks by the central bank can be either smoothed or amplified by commercial banks. A variable representing the credit channel may thus be hypothesized as being regime-shifting (see Galbraith (1996)). Secondly, the impact of monetary shocks on activity is also conditioned by the credibility of monetary policy. Financial variables such as interest rate differentials reflect the agents' expectations about future conditions of the business cycle. People may want to increase saving if they foresee a slowdown. In this case an expansive monetary policy might be ineffective. There is evidence in the literature that such behaviors induce asymmetric dynamics in the business cycle (see Aftalion (1997)). Other examples of regime-shift variables could be evoked: the indexing rules that characterize the wage-price loop, the pricing rules on the good markets, the growth rate of federal expenditures, the output-gap. Whatever the case, it seems difficult to assume that a money-output equation has parameters that are invariant across alternative values of the regime-shift variables. In this paper, we use an MRSTAR model to see whether the state dependent approach helps capturing the money non-neutrality on the business cycle. We proceed as follows.

Section 2 presents the MRSTAR model and motivates the choice of economic variables. Our model consists of several endogenous variables that we choose as indicators of the business cycle: the real GDP, aggregate consumption, employment and investment. We also consider an equation determining the inflation rate. The exogenous variables are, respectively, the growth rate of M1, a total productivity index variable, and the variation of the federal budget deficit. The regime-shift - or transition - variables that affect the shape of the business cycle are the output-gap, the wedge, and financial variables that include indicators of the credit channel and interest rate term structure variables.

Section 3 focuses on estimation and testing procedures of MRSTAR models. We firstly use a procedure for identifying nonlinear correlations between monetary (and financial) variables and real variables. This preliminary step is needed to assert that money is non neutral on real activity. Secondly, MRSTAR estimation and testing procedures are exposed and results for the US economy are obtained using quarterly data over the period 1975:1 to 1998:2. For each endogenous variables we identify four regimes in their cycles and study the intrinsic dynamics within each regime.

Section 4 presents simulation results on the impulse response functions. The monetary shocks are supposed to be unanticipated. These exhibit different dynamics from a regime to another, what may be interpreted as an evidence of asymmetric effects of monetary policy.
2 The money-output MRSTAR Model

Throughout the paper, the influence of monetary impulses on the real activity will be based on the propagation and impulse approach as initiated in earlier studies of the business cycle. As to the first point (the propagation mechanism), we describe the intrinsic structure of the economy using MRSTAR formulation. This allows us to discuss the nature of nonlinearity induced by a time-dependency of the structural parameters. Sections 2 and 3 introduce some basic methodological aspects of the MRSTAR models and examine estimation results for the US economy. The effects of monetary impulses are studied in section 4.

2.1 General formulation of a MRSTAR model

A MRSTAR model for a univariate time series $y_t$ can be formulated as follows (van Dijk and Franses (1999)):

$$ y_t = \hat{A}_1^0 w_t [1_i F_1(s_{1t}; \hat{o}_1; c_1)] + \hat{A}_2^0 w_t F_1(s_{1t}; \hat{o}_1; c_1) \left[ 1_i F_2(s_{2t}; \hat{o}_2; c_2) \right] + \hat{A}_3^0 w_t [1_i F_1(s_{1t}; \hat{o}_1; c_1)] + \hat{A}_4^0 w_t F_1(s_{1t}; \hat{o}_1; c_1) \ F_2(s_{2t}; \hat{o}_2; c_2) + \epsilon_t $$

(1)

where $\epsilon_t \sim i.i.d. (0, \sigma^2)$, $w_t = (1; y_{t-1}; \ldots; y_{t-p}; x_{t-1}; \ldots; x_{tk})^0$, $\hat{A}_i = (\hat{A}_{i,0}; \hat{A}_{i,1}; \ldots; \hat{A}_{i,m})^0$ for $i = 1; \ldots; 4$ and $m = p + k$. $F_1$ and $F_2$ are logistic functions given by

$$ F_i(s_{it}; \hat{o}_i; c_i) = \left[ 1 + \exp \left( i \ \hat{o}_i (s_{it} - c_i) \right) \right]^{-1} $$

(2)

where, for $i = 1; 2$, $\hat{o}_i$ and $c_i$ are scalars with $\hat{o}_i > 0$. The transition variables $s_{it}$ can be lagged endogenous variables or exogenous variables. The restrictions $\hat{o}_i > 0$ are identifying conditions. The slope parameters $\hat{o}_i$ are indicators of the speed of the transition between two extreme regimes and the $c_i$ are the half-way points between these regimes.

The model given by (2) and (3) generalizes the original LSTAR (Logistic STAR) model given as follows:

$$ y_t = \hat{A}_1^0 w_t [1_i F(s_{it}; \hat{o}_1; c)] + \hat{A}_2^0 w_t F(s_{it}; \hat{o}_1; c) + \epsilon_t $$

(3)

where $F$ is a logistic function given by

$$ F(s_{it}; \hat{o}_1; c) = \left[ 1 + \exp \left( i \ \hat{o}_1 (s_{it} - c) \right) \right]^{-1} $$

(4)

This last model is able to describe asymmetric behavior where two extreme regimes have different dynamics with a smooth transition from one.
to the other one. It may be noted that, when \( \theta > 1 \), the LSTAR model approaches a TAR (threshold autoregressive) model with two regimes (see Tong (1990)), and, when \( \theta = 0 \), it approaches a linear model. Another well-known STAR model is the ESTAR model (Exponential STAR) given by (??) and an exponential function \( F \) defined by

\[
F(s_t; \theta; c) = 1 + \exp \left( \frac{i}{\theta} (s_t - c)^2 \right)
\]

with \( \theta > 0 \), and representing a different dynamic behavior.

STAR models can only accommodate two regimes. MRSTAR models are thus introduced in van Dijk and Franses (1999) in order to take into account more than two regimes. More precisely, they allow detecting four distinct regimes, each corresponding to some extreme values of both logistic transition functions \( F_1 \) and \( F_2 \) that vary between 0 and 1:

\[
\begin{align*}
8 & \quad F_1 = F_2 = 0 : y_t = A_{01} w_t + \epsilon_t \\
F_1 & = 1 \text{ and } F_2 = 0 : y_t = A_{02} w_t + \epsilon_t \\
F_1 & = 0 \text{ and } F_2 = 1 : y_t = A_{03} w_t + \epsilon_t \\
F_1 & = F_2 = 1 : y_t = A_{04} w_t + \epsilon_t 
\end{align*}
\]

The model is therefore locally linear in \( w_t \). This approach could be generalized up to \( k \) distinct regimes. For our purpose, it is worthwhile noting that MRSTAR models can be considered as time-varying coefficient models - just as the STAR models-. This can be shown by rewriting the model (??) as follows:

\[
y_t = \frac{1}{4} w_t + \epsilon_t
\]

with

\[
\frac{1}{4} = A_{11} [1, F_1(s_{1t}; \theta_1, c_1)] + A_{12} F_1(s_{1t}; \theta_2, c_1) [1, F_2(s_{2t}; \theta_2, c_2)] + A_{21} [1, F_1(s_{1t}; \theta_2, c_1)] + A_{22} F_1(s_{1t}; \theta_2, c_1) F_2(s_{2t}; \theta_2, c_2)
\]

Therefore, the variability of parameters appears to be depending on both \( F_1 \) and \( F_2 \), and consequently nonlinearly on the transition variables \( s_{1t} \) and \( s_{2t} \). In other words, the coefficients for \( w_t \) change smoothly with \( s_{1t} \) and \( s_{2t} \).

2.2 Application to the money-output relationship

The nonlinear specifications that we introduce below can be seen as reduced forms of a nonlinear structural model linking money to real variables. We,
however, do not seek the ‘true’ structural model, but simply use equations that mimic the asymmetry of monetary policy over the business cycle when the switching of the economy between the different phases is governed by regime-shift or transition variables. This section motivates the choice of our variables (endogenous, exogenous and transition variables).

We use the following notations:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>Real GNP</td>
</tr>
<tr>
<td>I</td>
<td>Investment</td>
</tr>
<tr>
<td>M1</td>
<td>Monetary index M1</td>
</tr>
<tr>
<td>P</td>
<td>Price</td>
</tr>
<tr>
<td>DEF</td>
<td>Federal deficit</td>
</tr>
<tr>
<td>PTY</td>
<td>Productivity</td>
</tr>
<tr>
<td>L</td>
<td>Employment</td>
</tr>
<tr>
<td>SL3</td>
<td>3-year interest rate term structure</td>
</tr>
<tr>
<td>WEDGE</td>
<td>Wedge</td>
</tr>
<tr>
<td>SL10</td>
<td>10-year interest rate term structure</td>
</tr>
<tr>
<td>GAP</td>
<td>Output-gap</td>
</tr>
<tr>
<td>CC</td>
<td>Proxy of the credit channel</td>
</tr>
<tr>
<td>C</td>
<td>Consumption</td>
</tr>
</tbody>
</table>

We assume that $y$ is one of the following variables: $4 \text{GDP}, 4\text{L}, 4\text{I}, 4\text{P}, 4\text{C}$. The variables $x_1; \ldots; x_k$ in the vector $w$ are the exogenous variables: $4\text{DEF}, 4\text{M1}, \text{and} 4\text{PTY}$. We further choose $s_1$ and $s_2$ as contemporaneous or lagged observations of one of the following variables: $4\text{GAP}, 4\text{SL3}, 4\text{SL10}, 4\text{CC}, 4\text{WEDGE}$. $4$ denotes the first-difference operator that we apply to the variables -measured in logarithm except DEF; and GAP-.

Our MRSTAR model thus consists of $5$ equations-one for each endoge-
nous variable - with the following specification:

\[ 4 y_t = A_{10} + A_{11} 4 y_{t-1} + A_{21} 4 M_{1t} + \sum_{i=1}^{A_{31}} 4 DEF_{ti} k + \sum_{j=0}^{A_{41}} 4 PTY_{ti} j \]

\[ + B_{10} + B_{11} 4 y_{t-1} + B_{21} 4 M_{1t} + \sum_{i=1}^{B_{31}} 4 DEF_{ti} k + \sum_{j=0}^{B_{41}} 4 PTY_{ti} j \]

\[ + D_{10} + D_{11} 4 y_{t-1} + D_{21} 4 M_{1t} + \sum_{i=1}^{D_{31}} 4 DEF_{ti} k + \sum_{j=0}^{D_{41}} 4 PTY_{ti} j \]

\[ + H_{10} + H_{11} 4 y_{t-1} + H_{21} 4 M_{1t} + \sum_{i=1}^{H_{31}} 4 DEF_{ti} k + \sum_{j=0}^{H_{41}} 4 PTY_{ti} j \]

With such a model, we define four regimes that are delimited by the threshold parameters \( c_1 \) and \( c_2 \):

<table>
<thead>
<tr>
<th>( S_{2t} )</th>
<th>( S_{2t} &lt; c_2 )</th>
<th>( S_{2t} &gt; c_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{2t} &lt; c_1 )</td>
<td>Regime 1</td>
<td>Regime 2</td>
</tr>
<tr>
<td>( S_{2t} &lt; c_1 )</td>
<td>Regime 3</td>
<td>Regime 4</td>
</tr>
</tbody>
</table>

For illustration purpose, suppose that \( s_{2t} \) is a lagged observation of 4 GAP. \( c_1 \) is a threshold value of the output-gap variations, above which the economy is considered to experience a situation of great depression. Conversely, when 4 GAP is under \( c_1 \), the economy is assumed to evolve in a phase of recovery or expansion. Further, suppose that \( s_{2t} \) is a proxy of the credit channel. When this indicator takes a value under \( c_2 \), we assume that firms face credit rationing, while above this value the credit conditions are favorable for borrowing. Suppose that the endogenous variable \( y \) is 4 I (the
growth rate of investment). With such hypotheses, what would be the impact of a 1% increase in the money stock? Such an impact is captured by the elasticity coefficients $A_2j$, $B_2j$, $D_2j$, and $H_2j$, $j = 1; \ldots; q$. Intuitively, such a policy should have a small effect if, for instance, commercial banks smooth the initial variations of the money stock - which implies a credit rationing situation: here $s_{2t} < c_{2t}$, or if the economy is evolving in a situation close to the full-capacity utilization - here $s_{1t} < c_1$. Consequently, the coefficients of the money-investment equation should be smaller in regime 1 than in the three other regimes.

Now, we briefly motivate the choice of the variables included in our equations. Basically, our model must consist of, at least, two equations with $4 \text{GDP}$ and $4 \text{P}$ as the endogenous variables. Since the linear money-output and money-inflation equations are particular cases of MRSTAR equations, it also seems natural to choose the exogenous variables that are usually included in the linear specifications. Most often, one uses the contemporaneous and lagged values of:

- the money growth rate (here $4 \text{M}_{1t-j}$, $j = 1; \ldots; q$);
- the federal budget deficit (here $4 \text{DEF}_{tk}$, $k = 0; \ldots; r$);
- the treasury bill rate (as explained later, this variable is chosen here as a transition variable);
- the unexpected changes in the real price of energy (here, we use another proxy of the supply shocks: the variation of total productivity $4 \text{PTY}_{tl}$, $l = 1; \ldots; s$);
- the unemployment rate (such a variable is omitted here).

To capture the gradual and smooth changes in the impact of monetary shocks on output, a model with time-varying parameters is more suitable than the standard linear formulation (see the discussion before). In this view, we include transition variables in our equations. The effect of monetary policy can be more or less strong and induces sluggish adjustment of the output, depending on the transmission mechanisms of money shocks.

We first consider financial channels through the credit channel and the slope of the term structure of interest rates. The literature has indeed emphasized that the estimation of a money-output equation is sensitive to the choice of the variables that measure the monetary policy. The credit channel mechanism is consistent with, either the standard IS-LM model of output fluctuations - see Blinder (1987) - or with the microeconomic imperfections in the credit market - as suggested by Stiglitz and Weiss (1981). The proxy that we choose as a measure of the credit channel is measured by the difference between the federal reserve base rate and commercial banks loan rate. We further consider financial series that contain information about the monetary policy: the slope of two interest rate term structures - $4 \text{SL}_3$ and
that are based on the difference between the 3 and 10 year treasury bonds and the rate of 3 month treasury bills. The interest rate term structure can be upward or downward sloping, depending upon whether the monetary policy is expansive or restrictive. There are some empirical evidence in the literature that the difference between long-term and short-term interest rates is a good predictor of real activity (see Stock and Watson (1989) and Estrella and Hardouvelis (1991)). In what follows, $s_2$ will thus be a lagged value of one of the following variables: 4 SC, 4 SL3, 4 SL10. It has been shown in previous studies that they can cause asymmetric responses in the reaction of output and inflation to money shocks. This issue is discussed by Galbraith (1996) and Aftalion (1997) who use threshold and STAR models to capture the asymmetric effects of monetary policy. In this paper, we show that such effects also emerge in the more general setting of the MRSTAR models.

Aside from the financial transmission mechanisms, there are other reasons why the coefficients of an equation relating money to output and inflation may be time-varying. Recent papers show that short-term Phillips curves are nonlinear. Even though the exact form of nonlinearity still remain ambiguous, it is argued that the nonlinear form comes from the following property: the slope of Phillips curves is a function of present and past macroeconomic conditions (for a synthesis of theoretical arguments, the reader is referred to Dugasquier and Richets (1998) and Yates (1998)). The factors that condition the slope of the Phillips curve include:

- the economy capacity constraint,
- the inflation volatility,
- the individual firms' adjustment costs,
- firms' market power in the good market,
- nominal rigidities in wages and prices.

With these factors, Phillips curves are concave or convex, thereby inducing asymmetric effects of output to money supply variations. The most important consequence, perhaps, is that asymmetric responses to money shocks are associated with the timing of monetary policy. Nothing indeed guarantees that a large variation in the monetary instrument is equivalent to successive small changes. Precisely, if the central bank allows deviations from the target for some time, a larger movement in the monetary instrument will be needed to achieve the desired level. This is due to the fact that a very small initial change in monetary policy may be amplified in the course of time because the effects on output and inflation are time-dependent. Conversely, a large movement in monetary aggregates does not necessarily imply a large response of output and inflation, if the dynamic effects go through channels that smooth the initial impulse and induce sluggishness in response to shocks. The fact that outside money may have an impact through 'real channels' -and
not only through financial channels - has motivated recent empirical works. For instance, E-liason (1999) use a STAR model as a reduced form of the Phillips curve and shows that money does have asymmetric effects on output in Australia and Sweden, while such effects are not very conclusive for the United States. In this paper, we show that the same conclusion however holds for the US economy if one uses MRSTAR equations instead of STAR models. We stress that such 'real effects' of monetary policy exist, even outside the framework of real business cycles and even when the money supply is assumed not to be endogenous.

In view of the preceding remarks, the following transition variables are added to our equations - they are used in the definition $s_{1t}$. Firstly, we choose a variable that captures the effects of capacity constraints. If the short-run Phillips curve are indeed nonlinear, then the consequences of money shocks may be larger when the economy operates close to capacity. Thereby, excess demand and excess supply situations may induce asymmetric responses of the output and the inflation. The proxy that we choose to capture such effects is the variations of the output-gap, $4\, GAP$. Secondly, the effects of monetary impulses also depend on the microeconomic factors that render the response of inflation more or less sluggish. Shifts in the coefficients of the money-inflation equation are conditioned by the wage-price adjustment mechanisms and by the pricing rules in the good markets (for theoretical arguments relating the pricing behaviors to state-dependent relationships between money and output, the reader may refer to Dotsey et alii. (1999). Accordingly, we also consider the $WEDGE$ as a possible transition variable. To sum up, in the sequel $s_{1t}$ will be a lagged value of either $4\, GAP$, or $4\, WEDGE$.

More than output and inflation, we also consider the effects of monetary shocks on employment, investment and consumption. The reasons for doing so are twofold. Firstly, it is interesting to obtain results on several indicators of the business cycle, rather than selecting arbitrarily the output fluctuations. Secondly, not only do the transition variables modify the slope of the real GNP cycle, but they also affect the dynamics of its main determinants: here we restrict our attention to consumption and investment.

3 Estimation procedures and results for the US Economy

It seems unlikely that the specified MRSTAR equations will be able to predict the impact of monetary policy on real aggregates unless we provide some
'stylized facts' concerning the nonlinear correlations between financial variables and output, employment, consumption, investment and the inflation rate. The starting point thus consists in showing that such correlations do characterize the US business cycle. In doing so, we shall neither discuss the problem of causality, nor shall we debate on the nature of money - inside or outside money. Even though the endogenous money approach has been successfully used to study the impact of money on the real sector, we stick here to the Keynesian view and regard correlations between money and activity as reflecting the influence of monetary policy on real activity. We seek to examine the statistical comovements-movements of our series in a nonlinear setting. In this view, a two-step procedure may be suggested. Firstly, the observation that nonlinear components in time series are present in the higher order moments than the second, motivates the search for a method generalizing the usual linear correlation functions. We investigate such a method using the bicorrelation function proposed recently by Hinich (1996) and Brooks and Hinich (1997). Secondly, for modeling purpose, one must see whether a standard linear money-output model is rejected against a specific nonlinear model - here the MRSTAR model.

These issues are examined in the next section.

3.1 Testing for the presence of nonlinear relationships between money, financial variables and real variables

3.1.1 A nonlinear correlation coefficient

Several procedures for identifying nonlinear correlations in multivariate time series are now available in the literature. Brock et alii. (1990) propose a nonlinear Granger causality test that can be applied to the residuals of bivariate vector autoregressions. Priestley (1988) suggest an approach based on the cross-bispectrum. Recently, Hinich (1996) developed a test, easy to implement, where nonlinearities characterize the third-order cumulants of time series. We follow his approach here.

Heuristically, the arguments are the following. Consider a bivariate process \( fX_t, Y_t \) \( t=1 \), stationary up to the third-order. Let \( C_{X,Y}(r,s) \) be the coefficient of \( (z_r; z_s) \) in the expansion of the cross-cumulant generating function:

\[
K(z_r, z_s) = \ln \mathbb{E} [\exp(z_Y + z_r X_{t_5} + z_s X_{t_5})]]: \tag{8}
\]
Then,

\[ C_{XXY}(r; s) = \mathbb{E}[(Y_t i \gamma)(X_{t+r} i \gamma)(X_{t+s} i \gamma)] \]

is called the third-order cross-cumulant of \( fX_t; Y_t g_{t=1}^T \). \( C_{XXY} \) generalizes the definition of the standard second-order cross-cumulant:

\[ C_{XY}(s) = \mathbb{E}[(Y_t i \gamma)(X_{t+s} i \gamma)] \]  \hspace{1cm} (9)

The main difference between a linear and a nonlinear process can now be stated in the following terms. Assume that in the bivariate process \( fX_t; Y_t g_{t=1}^T \), \( fX_t g_{t=1}^T \) is the input and \( fY_t g_{t=1}^T \) is the output. Suppose further that \( X_t = Y_t = 0 \). The discrete cross-bispectrum is defined as:

\[ B_{XXY}(\nu_1; \nu_2) = \frac{\mu_1 \mu_2}{2^{1/4}} \sum_{r=i_1} C_{XXY}(r; s) \exp \left[ i (\nu_1 r + \nu_2 s) \right] \]  \hspace{1cm} (10)

where \( i \frac{1}{4} \) \( j \frac{1}{4} \) \( j = 1; 2 \). Such a Fourier transform exists if we assume that

\[ jC_{XXY}(r; s)j < +1 \]  \hspace{1cm} (11)

The cross-bispectrum may also be written as

\[ B_{XXY}(\nu_1; \nu_2) d\nu_1 d\nu_2 = \mathbb{E}[dZ_X(\nu_1) dZ_X(\nu_2) dZ_Y(\nu_1 + \nu_2)] \]  \hspace{1cm} (12)

where \( dZ_X(\nu_1) \) is a complex stochastic orthogonal increment process that describes the contribution of the components with frequencies \( \nu_1 \) \( j = 1; 2 \) in \( fX_t g_{t=1}^T \) to the component with frequency \( \nu_1 + \nu_2 \) in \( fY_t g_{t=1}^T \). Suppose for instance that the input consists of two sine waves:

\[ X_t = A_1 \exp[i \nu_1 t] + A_2 \exp[i \nu_2 t] \]  \hspace{1cm} (13)

Then, the output will consist of the input frequencies with their harmonics plus the sum of the input frequencies and their harmonics. Such a property is typical of nonlinear systems where ‘frequency multiplication’ and ‘harmonic distortion’ occur. Conversely, in a linear system we would observe a sine wave of the same frequency in the input and output variables (with a scaled amplitude and a shift in the phase in the output). Furthermore, the principle of superposition in linear models implies that the total output is the sum of the outputs corresponding to the individual inputs.
The test of nonlinear correlation suggested by Hinich (1996) relies on the preceding ideas, but, instead of the frequency domain, the author chooses a temporal domain approach. His aim is to study the dependence of $Y_t \sigma_{Y_t}^2$ to variations in $X_t \sigma_{X_t}^2$ that occurred at different time periods in the past. Since the cross-bispectrum is the frequency domain analog of the third-order cross-cumulant, the testing procedure amounts to see whether $C_{XXY}(r; s)$ varies across pairs of lags $(r; s)$.

Under the null hypothesis of linearity, $C_{XXY}(r; s) = 0$ for all pairs $(r; s)$, except when $r = s = 0$. Under the alternative hypothesis of nonlinearity, $C_{XXY}(r; s) \neq 0$ for at least one pair $(r; s)$ with $r \neq 0, s \neq 0$. So the test procedure is as follows:

$$H_0 : C_{XXY}(r; s) = 0; \; \forall (r; s);$$

against

$$H_1 : C_{XXY}(r; s) \neq 0; \; \forall (r; s);$$

The test is analog to a test for symmetry in multivariate analysis (see Anderson (1958)). A global test statistic is

$$H_{XXY}(N) = \frac{p}{N} \sum_{r=1}^{m} \sum_{s=1}^{L} [C(r; s)]^2 \text{ with } s \neq 1; 0; 1 \quad (14)$$

$m = \max(r; s), L = N^c$ with $0 < c < 1 = 2$. It has been shown that this statistic is asymptotically distributed as a Normal variate, provided that the original series are standardized to have a zero mean and a unit variance (see Brooks and Hinich (1997)).

3.1.2 The results

The data are described in Appendix 1. We take the first difference of all the variables in order to deal with non-stationarity problems. To compute the statistic $H_{XXY}(r; s)$ we use the residuals of Box-Jenkins and VAR models of our variables. The table 1 below indicates the values of $H_{XXY}(r; s)$ for the hypothesis that the lags of the exogenous variables are linearly correlated.

---

2 Results concerning stationarity tests (ADF, PP, KPSS) are available upon request to authors. A more rigorous approach would consist in applying specific non-stationarity tests to nonlinear systems. We, however, neglect such an issue here but this opens a new agenda for a future research. Moreover, since our analysis concerns the asymmetry of monetary shocks over the business cycle, it seems natural to work with the first-order differences of logarithmic variables.
with the contemporaneous observation of the endogenous variables. This table contains the results when the endogenous variables are, respectively, 4 GDP, 4 CONS, 4 l, 4 L and 4 P. The exogenous variables are in rows. The numbers in tables must be compared to 1.96. The null hypothesis of linear correlation is rejected if $H_{XY}(r; s) > 1.96$.

Following Brooks et alii (1997), we choose $c = 0.4$. This choice is based on the results of Monte Carlo simulations and is recommended in order to maximize the power of the test whilst ensuring a valid approximation to the asymptotic theory. From the table 1, it is seen that all the values are greater than 1.96, which indicates a strong evidence in favor of nonlinear relationships between series.

Table 1. $H_{XY}(r; s)$ estimates on the residuals of ARMA models

<table>
<thead>
<tr>
<th></th>
<th>4 GDP</th>
<th>4 CONS</th>
<th>4 L</th>
<th>4 P</th>
<th>4 I</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>8.3028</td>
<td>11.5288</td>
<td>7.2602</td>
<td>9.9290</td>
<td>11.6269</td>
</tr>
<tr>
<td>PTY</td>
<td>23.0742</td>
<td>18.9117</td>
<td>15.5566</td>
<td>14.6662</td>
<td>13.5208</td>
</tr>
<tr>
<td>DEF</td>
<td>5.9074</td>
<td>8.2249</td>
<td>6.5509</td>
<td>9.3332</td>
<td>7.0336</td>
</tr>
<tr>
<td>CC</td>
<td>21.8794</td>
<td>13.7605</td>
<td>22.6709</td>
<td>12.4923</td>
<td>17.1341</td>
</tr>
<tr>
<td>SL3</td>
<td>18.6263</td>
<td>17.6656</td>
<td>16.8343</td>
<td>8.8743</td>
<td>9.6488</td>
</tr>
<tr>
<td>SL10</td>
<td>25.2736</td>
<td>24.989</td>
<td>17.6540</td>
<td>15.3109</td>
<td>16.7913</td>
</tr>
</tbody>
</table>

There seems to be a prominent evidence that monetary and financial aggregates do non-linearly affect real variables over the business cycle. This leads us to argue that the use of nonlinear framework when studying the money-output relationship may substantially modifies some of the conclusions traditionally obtained in linear contexts. There is today a tendency among economists to regard the linear correlation between money and output as reflecting causality from output to money (to say it another way, it is the money demand that responds to output variations). This viewpoint is largely debated in the RBC literature. In such a context, the money base is usually uncorrelated with real variables. In the tables above, the consideration of CC, SL3, and SL10 help us disentangling demand and supply shocks in M1. These variables indeed capture - in our nonlinear framework - the contribution of supply shocks to movements in aggregate money. From table 1, we see that the lagged values of these variables are highly non-linearly correlated with the contemporaneous values of the endogenous variables. This raises the following question. Which of the financial variables appears
to dominate money as a forecaster of real activity? To answer this question, it is worth noting that the higher the value of the statistic $H_{XYY}(r;s)$ is, the smaller the value of the marginal significance that would indicate a rejection of the null hypothesis will be. In other terms, the importance of the row variables for predicting the column variables increases with the value of $H_{XYY}(r;s)$. As is seen, even though $M_1$ may be though as having a predictive power, the following observations can be made: the slope of the term structure $SL_{10}$ is the best predictive monetary variable for $GDP$, $C$, and $P$, while the credit channel $CC$ dominates for $L$ and $L$.

Apart from the considerations above, we also note the nonlinear correlation between the real activity and the two other transition variables $GAP$ and $WEDGE$. These results may be illustrative of nonlinear Phillips curves as argued in section 2.2.

3.2 Specification of MRSTAR models

The test discussed in the last paragraph is a ‘Portmanteau’ test, in the sense that it does not make any distinction between the nonlinear processes that underlie the relationships between monetary variables and real variables. We now go a step further by testing for the presence of specific nonlinearities. In view of the preceding discussions, we focus on MRSTAR models.

3.2.1 Presentation of the method

When testing for the existence of MRSTAR models, it is firstly useful to see whether we could model the links between our series by using STAR specifications. The reason for doing this is simply that STAR models are particular cases of MRSTAR models. So, we would not expect to find a process with two transition variables, if there is not an underlying model to our data with, at least, one transition variable. The modelling cycle for building a MRSTAR model thus consists of three stages, the two first ones corresponding to the building and estimation of a STAR model (see Granger and Teräsvirta (1993) and Teräsvirta (1994) for instance), the last one, specific to the multiple regimes specification, being described in van Dijk and Franses (1999):

---

3Even though we explore an interpretation here in terms of causality, it is worth remembering that we are not using a proper test of causality. Further research would be to employ here a Granger non causality test developed in a nonlinear framework, see for instance Peguin-Feissolle and Teräsvirta (1999). So, our arguments may be criticized. Furthermore, there can be several explanations to the observed correlation. For instance, outside money may not cause real effects, but may be correlated with the GDP and employment because the FED includes these variables in its reaction function.
- step 1: one rstly test for the presence of a linear AR process against the alternative of a STAR model;
- step 2: if the null hypothesis is rejected, then it becomes necessary to distinguish between LSTAR and ESTAR models. This second step implies that one tries to discriminate between the transition functions that are the most used, namely the logistic function (??) and the exponential function (??);
- step 3: the third step involves the testing of LSTAR and ESTAR specifications against the alternative of MRSTAR (with a logistic or an exponential transition function depending on the choice made at the second step).

The preceding steps, however, imply some difficulties because the LSTAR and MRSTAR model include nuisance parameters that are not identified under the null hypothesis of linearity. Several procedures have been suggested in the literature to cope with this problem. We adopt here an approach - exposed in Teräsvirta (1994) and adapted to MRSTAR by Dijk and Franses (1999) - where Taylor expansions of the nonlinear functions are used to form Lagrange Multiplier tests. We are going now to precise the different steps seen above.

Step 1 involves two stages:
(a) We specify a linear autoregressive model by using a selection procedure such as Akaike information criteria accompanied by a proper test for residual autocorrelation; this last test is important because omitted autocorrelation may cause rejection of the linearity hypothesis. At the end of this stage, we know \( w_t \), the \( m \times 1 \) vector of explicative variables.
(b) We test now linearity for different values of the transition variable \( s_t \). After the estimation of the linear model determined above, linearity is tested against the alternative of a STAR model. If linearity is rejected against a STAR model for more than one transition variable, we choose the variable giving the lowest \( p \) value to be the transition variable, i.e. giving the strongest rejection of the linearity hypothesis. We use the LM-type test developed by Lüükkonen, Saikkonen and Teräsvirta (1988) where the auxiliary regression is given by:

\[
\mathbf{b}_t = -\bar{\gamma}_1 w_t - \hat{\bar{\gamma}}_2 s_t - \hat{\bar{\gamma}}_3 s_t^2 - \hat{\bar{\gamma}}_4 s_t^3 + \mathbf{1}_t
\]

where \( \mathbf{b}_t \) is the residual of the linear model. The null hypothesis is \( H_0: \bar{\gamma}_2 = \bar{\gamma}_3 = \bar{\gamma}_4 = 0 \) against the alternative hypothesis that \( \gamma_j \neq 0 \)

For a detailed exposition of the testing procedures used for STAR models, the reader is referred to Granger and Teräsvirta (1993) and Teräsvirta (1994)
for at least one $j$, where the $\bar{\eta}_j$ are $m \leq 1$ vectors. The procedure is carried out for several transition variables.

Step 2 consists in choosing between LSTAR and ESTAR model by using a sequence of nested hypothesis given by:

$$H_{01}: \bar{\eta}_4 = 0$$
$$H_{02}: \bar{\eta}_3 = 0 / \bar{\eta}_4 = 0$$
$$H_{03}: \bar{\eta}_2 = 0 / \bar{\eta}_3 = \bar{\eta}_4 = 0$$

The rejection of $H_{01}$ implies the selection of a LSTAR model. If we accept $H_{01}$ and reject $H_{02}$, we choose a ESTAR model. The acceptation of $H_{01}$ and $H_{02}$ and the rejection of $H_{03}$ leads to a LSTAR model.

Granger and Teräsvirta (1993) and Teräsvirta (1994) recommend to choose the STAR model on the basis of the lowest $p_1$ value: if the $p_1$ value of $H_{02}$ is the smallest of the three, select an ESTAR model; if not, choose a LSTAR model. Assume at this point that the chosen model is a LSTAR model.

In step 3, we test the hypotheses of STAR specifications against the alternative of MRSTAR models; it is convenient to rewrite the equation (15) as follows:

$$y_t = A_{11}^* w_t + A_{21}^* w_t F_1(s_{1t}; \theta_1; c_1) + A_{31}^* w_t F_2(s_{2t}; \theta_2; c_2) + \bar{\epsilon}_t$$

with $A_{11}^* = A_1^* A_2^* A_3^* \bar{A}_4$. Suppose that, in step 2, the model selected was a LSTAR model - so that the problem is now to test the null hypothesis of a LSTAR model against the alternative of a MRSTAR model. $F_2(s_{2t}; \theta_2; c_2)$ is replaced with a third-order Taylor expansion in the neighborhood of $\theta_2(s_{2t}; c_2) = 0$ in order to obtain, after rearranging the different terms, the following model:

$$y_t = \mu_0 w_t + \mu_1 w_t F_1(s_{1t}; \theta_1; c_1) + -\gamma_0 w_t s_{2t} + -\gamma_2 w_t s_{2t} + -\gamma_3 w_t s_{2t} + -\gamma_4 w_t s_{2t} + -\gamma_5 w_t s_{2t} + -\gamma_6 w_t s_{2t} + \bar{\epsilon}_t$$

where the parameters vectors $\bar{\eta}_i$ are functions of the $\bar{A}_i^*$. The null hypothesis can be written as $H_0^n: \bar{\eta}_i = 0, i = 1; \ldots; 6$ and is tested by using again a LM-type test statistic as described by Dijk and Franses (1999). The same rules as above apply (in case of several transition variables $s_{2t}$, we choose the variable that gives the statistic with the lowest $p_1$ value).
3.2.2 The results

The results on U.S. data are reported in Appendix 2. We first give the estimates of the linear models by ordinary least squares together with diagnostic statistics in Appendix 2.1. The specifications are obtained using Akaike information criteria and a test for residual autocorrelation. The p-values reject serial correlation and the presence of conditional heteroskedasticity and of heteroskedasticity in the residuals for all equations.

Table A.2.2.1 shows the p ¡ values of the LM test statistics when the hypothesis of linearity is tested against the alternative of STAR models for different values of the transition variables, i.e. the contemporaneous or lagged observations of one of the following variables: 4 GAP, 4 SL 3, 4 SL 10, 4 CC, 4 WEDGE; the delay parameter is on the range 0 to 4. If linearity is rejected for more than one variable, we choose the variable giving the lowest p ¡ value; these transition variables are indicated in bold in the table. The test succeeds to reject linearity for each equation. We thus choose as transition variables

\[
\begin{align*}
&\text{\(\epsilon\) GAP}_t \quad \text{in the equation determining \(\epsilon\) GDP}_t \\
&\text{\(\epsilon\) GAP}_{t1} \quad " \quad \text{\(\epsilon\) C}_t \\
&\text{\(\epsilon\) CC}_{t1} \quad " \quad \text{\(\epsilon\) I}_t \\
&\text{\(\epsilon\) WEDGE}_{t1} \quad " \quad \text{\(\epsilon\) L}_t \\
&\text{\(\epsilon\) SL10}_{t1} \quad " \quad \text{\(\epsilon\) P}_t
\end{align*}
\]

Table A.2.2.2. contains the p ¡ values of the LM test statistics used to discriminate between LSTAR and ESTAR models. It is interesting to note that all the endogenous variables, except \(\epsilon\) P, are specified as LSTAR models, showing that expansion and contraction phases may have different dynamics. For \(\epsilon\) P, the specification as an ESTAR model did not give any significant improvement as compared to a LSTAR estimation. We thus decided to choose a LSTAR specification for the \(\epsilon\) P equation too.

Appendix 2.3 shows the results of the LSTAR models estimation for each equation, together with the diagnostic tests in order to check their robustness. It may be pointed out that estimating LSTAR and MRSTAR models by nonlinear least squares is not always easy; for instance, the sequence of estimates for the parameter \(\delta\) in (**) may converge rather slowly. Following Teräsvirta (1994), we scale the exponent of the transition function, \(F\), given by

\[
F (s_t; \delta; c) = [1 + \exp (i (\delta (s_t; c)))]^{\frac{1}{i}};
\]

by dividing it by the empirical standard deviation \(\delta_{\text{thres}}\) of the corresponding transition variable as follows:

\[
F (s_t; \delta; c) = [1 + \exp (i (\delta (s_t; c))/ \delta_{\text{thres}})]^{\frac{1}{i}};
\]

19
This allows making \( \frac{b^2_{\text{LSTAR}}}{b^2_{\text{Lin}}} \) scale-free and yields easier interpretations of its estimate. The variance ratios \( \frac{b^2_{\text{LSTAR}}}{b^2_{\text{Lin}}} \) give an idea of the relative gain in the \( \ldots \) from using a LSTAR model instead a linear model; they suggest here that the gain is always good, from 0.6479 for \( \xi \text{ GDP} \) to 0.9571 for \( \xi \text{ L} \). The appendix 2.3. shows that the null hypotheses of no autocorrelation can not be rejected in each equation (statistics \( \text{ET} \)); this is reinforced by the results of the statistics \( \text{ET} \) representing the LM test of serial independence introduced in Eitrheim and Teräsvirta (1996) for STAR models. Moreover, the models pass the test of no heteroskedasticity (statistics \( \text{White} \)). The same conclusion holds for the test of no autoregressive conditional heteroskedasticity (statistic \( \text{ARCH} \)).

Tables A.2.2.3 and A.2.2.4. report results of the \( F \)-version of the tests of LSTAR models against MRSTAR alternatives, on the basis of the estimated LSTAR models above. We can see the \( p \) values of the tests for various choices of the second transition variable \( s_{2t} \). The convention we adopt here is to consider as possible second transition variables the variables not already chosen in the preceding step as the \( \ldots \)rst transition variable \( s_{1t} \). Furthermore, the residuals estimated in the LSTAR models may not be exactly orthogonal to the gradient matrix used in the standard test LSTAR/MRSTAR given in Table A.2.2.3. and the partial derivatives of the \( \ldots \)rst transition function with respect to \( \theta_1 \) and \( c_1 \) in (??) may approach zero functions; van Dijk and Franses (1999) thus introduce a modified version of the test LSTAR/MRSTAR which is given in Table A.2.2.4. Both tables suggest that there is some evidence for considering the multiple regimes version of STAR modeling for all the models and give the transition variables in the second transition functions. It is worth mentioning here that the choice of \( \xi \text{ CC}_{ti} \) as second transition variable for the equation determining \( \xi \text{ GDP} \) didn’t give valuable results; therefore \( \xi \text{ SL}_{10t} \) will be used as transition variable. Finally, we choose the following second transition variables in the different MRSTAR models:

\[
\begin{align*}
\xi \text{ SL}_{10t} & \quad \text{in the equation determining } \xi \text{ GDP}_t \\
\xi \text{ SL}_{10t} & \quad \text{"} \\
\xi \text{ SL}_{3t} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\xi \text{ GAP}_{ti} & \quad \text{"} \\
\end{align*}
\]

Appendix 2.4. shows the estimation of the \( \ldots \)ve MRSTAR equations using the method of nonlinear least squares. To study the robustness checks of the estimations, we test the usual hypotheses on the residuals (no serial autocorrelation, no autoregressive conditional heteroskedasticity,\ldots). It is worth
mentioning that for the same reasons expounded before, we scale the transition functions \( F_1 \) and \( F_2 \) by dividing their expression respectively by \( \frac{1}{\text{thres}} \) and \( \frac{1}{\text{thres}} \), the empirical standard deviations of the transition variables, \( s_{1t} \) and \( s_{2t} \). Several conclusions can be drawn from the results. The null hypothesis of no serial correlation can not be rejected at 5% level for each equation, as shown by statistics \( GB \). The hypothesis of no heteroskedasticity (statistics \( \text{ARCH} \)) is not rejected, as well as the null hypothesis of no autoregressive conditional heteroskedasticity (statistics \( \text{ARCH} \)). Moreover, the relative gain in the \( \Delta \) from using a \( \text{MRSTAR} \) model instead a linear model, characterized by \( \frac{\Delta b_{\text{MRSTR}} - \Delta b_{\text{Lin}}}{\Delta b_{\text{Lin}}} \), is important in each equation, from 0.4827 for the equation of \( \zeta \ L \) to 0.7973 for \( \zeta \ P \), and more interesting than the \( \Delta \) from using a simple \( \text{LSTAR} \) model.

It is usually difficult to interpret the individual coefficients of the \( \text{MRSTAR} \) models. However, they provide useful information for understanding their dynamics. Firstly, the characteristic roots corresponding to each regime describe the nature of local dynamics in phases of recessions and expansions (see appendix 2.5). It is seen that, in many cases, the local dynamics is explosive. Indeed, the most prominent characteristic roots have a modulus greater than 1. We, however, observe that 4 GDP is ‘globally’ less volatile than the other variables (we have only one value above 1). Secondly, we add some figures (appendix 3). Figures 1a, 2a, 3a, 4a 5a show the predicted values of the endogenous variables from both linear models and \( \text{MRSTAR} \) models. Figures 1b, 2b, 3b, 4b, 5b show the shape of the \( \Delta \)st transition functions ordered over time and over the \( \Delta \)st transition variables; Figures 1c, 2c, 3c, 4c, 5c show the same curves for the second transition functions. Figures 1d, 2d, 3d, 4d, 5d depict the distributions of the observations of the endogenous variables across the different regimes.

When analyzing these figures, it can be noted that the \( \text{MRSTAR} \) models describe the most ‘turbulent’ periods in the data, better than the linear models. The periods of ‘explosive’ growth rates are related to values of \( b_1 \) and/or \( b_2 \) close to 1. For instance, we distinctly see the aftermath of the second oil crisis in 1979 (see figures 1a, 2a, 3a and figures 1c, 2b). It must be noted that no regime changes seem to occur after 1990:1. This corroborates the observations on the US business cycle.

The shape of the transition functions depends on the values of the estimated parameters \( b_1 \) and \( b_2 \) that indicate how rapid the transition from zero to unity is. Comparing figures 1b and 1c, it is seen that the smoothness of the transition from phases of expansion to phases of depression (or

\footnote{It is important to remark here that the regimes in question don’t correspond to the four extreme regimes considered in (??) but to the regimes described in the section 2.2.}
vice versa) is due to the second transition variable $4 \text{ SL}10$: The slope of the term structure thus renders the realization of the output growth rate smoothly cyclical. In 4 GDP equation, we obtain $b_2 = j 5.6\%$ (appendix 2.4). T hereby, for the period considered, an increase of the short-term interest rate above the long-term interest rate higher than 5.6\% is recessionary (the recessions correspond to those points in figure 1d located in regimes 1 and 3. The recessions occur after a delay of one quarter (the transition variable is $4 \text{ SL}10_{t-1}$). Such a delay may be attributable to inflation expectations that are central to the term structure transmission mechanism. Let us consider now the effect of the output-gap on the dynamic properties of 4 GDP. We have $b_2 = j 51.04\%$. Thus, a decrease of more than 51.04\% of current production under the potential production induces a depression. This happens abruptly. Indeed, the shape of $\Phi_1$ in figure 1b is a straight line. Contraction phases due to the effect of the output-gap are illustrated by points located in regimes 1 and 2 in figure 1d. However, if we look at the distribution of observations, we see that a broad range of them are located in the right-hand side of the vertical line, thereby suggesting that the role of the output-gap in explaining recessions over the period considered is less important than the role played by the interest rate term structure.

In the equations of 4 C and 4 P, the transition variables are the same as those of 4 GDP. The values of the threshold parameters are, however, different. For 4 C, the threshold value of 4 GAP is $j 6.91\%$ and the threshold value of 4 SL10 is 7.37\%. Further, both the estimated transition functions $\Phi_2$ and $\Phi_2$ have a smooth slope. When the long-term interest rate is increased above the short-term interest rate by more than 7.37\%, the aggregate consumption decreases. The increase of long-term interest rate corresponds to a decrease in the price of assets and this induces capital losses. Movements in the long-term interest rates thereby induces wealth effects. From figure 2d, we however may not expect 4 SL10 to play a major role in the growth rate of aggregate consumption over the period considered. Indeed, very few points are located in the regions corresponding to regimes 2 and 4. The distribution among the different regimes is better explained by the vertical line that corresponds (e.g. by 4 GAP). Figure 5d suggests that both 4 SL10 and 4 GAP explain the distribution of the observations corresponding to 4 P in the different regimes. The estimated threshold values are respectively 5.85\% for 4 GAP and 0.03\% for 4 SL10. Further, there are longer delays on the transition variables than in the equation of 4 GDP (the delays are two quarters for the output-gap and three quarters for the term structure). Inflationary periods occur when current production lies above its potential level. The benchmark value above which prices begin to increase is 5.85\%.

For the other variables, the aggregate investment and the employment,
we see in the estimations given in appendix 2.4 that all the coefficients corresponding to $4 \ M \ 1$ are not statistically significant. This would mean that outside money has no influence on the dynamics $4 \ I$ and $4 \ L$. This finding is in accordance with the arguments according to which only aggregate supply shock affects those variables. In figures 3d and 4d, we see that none of the transition variables $4 \ C$ and $4 \ SL \ 3$ are satisfactory in explaining the distribution of the points across the regimes. When outside money is increased, the Phillips curve may be difficult to exploit because workers renegotiate wage contracts and firms react by applying a markup to their labor costs. In such a context, the employment is not stimulated. This may explain why the coefficients on the variable $4 \ M \ 1$ are not significant.

4  Measuring the effects of monetary shocks: generalized impulse response functions

We now extend the MRSTAR model by introducing the impulse problem. In the first place, it is worth explaining how the monetary policy does take place. We shall consider that non-anticipated monetary impulses are caused by the unexpected variations of the short term interest rates by the central bank. The initiating swinging movements of the activity are studied throughout generalized impulse response functions.

4.1  The impulse problem

It is usually assumed in the literature that the central banks set the short-term interest rate according to a reaction function (the Taylor rule):

$$r_t = a \ P_t + b \ GAP_t$$

(19)

where $r_t$ is the short-term interest rate and $a; b$ are weights on the inflation rate and on the output level. Further, the aggregate supply of the economy is described by the following equation:

$$4 \ P_t = \sum_{i=0}^{X} \ A_i \ GAP_{t_1}$$

(20)

By combining both these equations, we obtain an equation relating the contemporaneous value of the short-term interest rate and the lagged values of the output-gap:
The latter equation can be used as a basis for forecasts of the future short-term interest rate. This leads to the following expression:

$$E[r_{t+j}] = \sum_{i=0}^{j} \mu_i \text{GAP}_{t+i} + X$$

Also, the short-term interest rate forecasts are usually described as a weighted sum of the contemporaneous short-term interest rate and long-term interest rates ($l_t$):

$$E[r_{t+j}] = \sum_{j=1}^{\infty} r_t + (1_{1 \leq j \leq \infty})$$

The combination of the last two equations yields the following expression:

$$l_t r_t = A_0 + \sum_{i=0}^{j} \mu_i \text{GDP}_{t+i} + \sum_{i=j+1}^{\infty} \mu_i \text{P}_{t+i} + \varepsilon_t$$

The term structure is thus a predictor of future output growth. Assuming that expectations of future growth rate are obtained by extrapolating past growth rates, we would obtain an equation relating the term structure to past growth rates.

Another remark is in order here. It is often argued that the term structure contains information about the credibility of monetary policy, notably trough the expectations of future inflation rates. Assuming again adaptive expectations, we would obtain an equation relating the term structure to past inflation rates.

The preceding remarks have some implications for our study of monetary policy. We estimate an equation that relates $\zeta \text{SL10}_t$ to the lagged values of $\zeta \text{GDP}$ and $\zeta \text{P}$:

$$\zeta \text{SL10}_t = \sum_{j=1}^{K} \mu_j \zeta \text{GDP}_{t+j} + \sum_{k=1}^{K} \hat{A}_{k} \zeta \text{P}_{t+k} + \varepsilon_t$$

where $\varepsilon_t \sim iid(0; \sigma^2)$. $\zeta \text{GDP}$ in the equation above can be replaced by any leading indicator of the business cycle, for instance $\zeta C$ (we envisage such a change when simulating the consumption equation). Given the state-dependency property of the MRSTAR models, it is worth noting that our formulation of the impulse mechanism implies feedback effects between the endogenous variables and the transition variable $\zeta \text{SL10}$. As we shall see, these feedbacks cause persistent dynamics in the responses to monetary shocks.
4.2 Monetary shocks and generalized impulse response functions

An impulse response function (IRF) measures the time profile of the effect of a shock on the behavior of a time series. This notion is very similar to the standard notion of ‘multiplier’. The main difference is that the analysis is done with respect to shocks or innovations rather than to the series themselves.

Following Koop et alii (1996), let us consider the time series model:

\[ Y_t = F (Y_{t-1}; \ldots; Y_{t-p}) + H_t V_t \]  

where \( F \) is a known function, \( Y_t \) is a \( K \times 1 \) random vector, \( V_t \) is a \( K \times 1 \) vector of iid random perturbations with zero means and finite variances, \( H_t \) is a \( K \times K \) random matrix which is a function of \((Y_{t-1}; \ldots; Y_{t-p})\): Upper-case letters design random variables and lower-case letters denote realizations of these random variables. In addition, we use the following notations: \( \mathcal{I}_{t-1} \) is the information set used to forecast \( Y_t \); \( !_{t-1} \) is a particular realization of \( \mathcal{I}_{t-1} \).

The usual impulse response function (IRF) is defined as the difference between two realizations of \( Y_{t+n} \) which are similar up to \( t-1 \). The first realization is such that a unique shock of size \( \pm \alpha \) affects the system between \( t \) and \( t+n \). The second realization, which is taken as the benchmark, assumes that the system is not hit by any shock between \( t \) and \( t+n \). We define the IRF as follows:

\[
\begin{align*}
\text{IRF}(n; \pm !_{t-1}) &= E [Y_{t+n} | V_t = \pm V_{t+1} = \ldots = V_{t+n} = 0; !_{t-1}] \quad \text{for } n = 1; 2; 3; \ldots \\
&= E [Y_{t+n} | V_t = V_{t+1} = \ldots = V_{t+n} = 0; !_{t-1}]
\end{align*}
\]

For nonlinear models, the IRF generally depends upon \( !_{t-1} \), the particular history chosen as the basis for comparison of the two realizations. It also depends on the sign and the size of the shock \( \pm \alpha \). Potter (1995) reports that, in this context, asymmetric responses occur in two main forms. Firstly, for any particular history, the effect of shocks of varying sizes and signs is not a simple scaling of a unit shock. Secondly, for the same shock but different histories, the response can differ markedly. In order to highlight these properties, it is more suitable to use a Generalized Impulse Response Function (GIRF). The GIRF uses the concept of expectation operator conditioned on the history and/or shock. The response is thus an average of what might happen given the past and the present. More formally, the GIRF in the case
of an arbitrary current shock \( v_t \) and history \( ! t; 1 \) is given by\(^6\):

\[
\text{GIRF}(n; v_t; ! t; 1) = E[Y_{t+n}|v_t; ! t; 1] - E[Y_{t+n}|! t; 1]
\]

for \( n = 0; 1; \ldots \). We see that the GIRF is the difference between two conditional expectations which are themselves random variables: GIRF \((n; v_t; ! t; 1)\) represents a realization of this random variable. We now describe how we compute the GIRF for the estimated MRSTAR models. Our simulations include the following steps.

**Step 1. Creating vectors of monetary impulses**

To study the impact of monetary policy on the real activity, we introduce monetary innovations in the estimated MRSTAR models. Our monetary innovations are computed as the residuals of equation (??). Let \( b_t \) be the series of estimated residuals, where \( b_t \sim N(0; b\sigma^2) \). Define \( b; \mu_j; \theta_k \) as the unbiased estimators of \( \mu_j; \theta_k \) in (??), and \( b\sigma^2; b\mu_j; b\theta_k \) as their estimated variance. These estimators are Normal variates.

Since the GIRF is a random variable, we need vectors of different monetary shocks. For each \( t \), define

\[
\tilde{\xi}_t = (\xi_{1t}; \xi_{2t}; \ldots; \xi_{Rt})
\]

We construct \( \tilde{\xi}_t \) as follows. We generate \((\tilde{\xi} + p + 1)R\) parameters

\[
(\mu_{1j}; \ldots; \mu_{Rj}; \theta_{1k}; \ldots; \theta_{RK});
\]

\( j = 1; \ldots; \tilde{\xi} \) and \( k = 1; \ldots; K \). These parameters are randomly sampled from Normal laws \( N(\mu_j; \mu_j; \theta_k; \theta_k) \); \( \mu_j; \theta_k \). From (??), we construct \( \xi_{rt} \) (\( r = 1; \ldots; R \)) as follows:

\[
\xi_{rt} = \xi_{1t}^A \mu_{1j} 4 GDP_{ti} j + \xi_{1t}^A \theta_{1k} 4 P_{ti} k
\]

**Step 2. Definition of forecasts of the endogenous variables without shocks**

We iterate the system of the following equations to obtain forecasts of the endogenous variables \( \xi \) GDP, \( \xi \) P, and \( \xi \) C (which are represented here by
the variable $\xi y$). We denote the forecasts $\xi y_{t+1}, \xi y_{t+2}, \ldots, \xi y_{T}$ where $r = 1, \ldots, R$, $t = \zeta + 1, \ldots, N$ and $N < T$. $\zeta$ is the maximum lag in the regressors of the estimated MRSTAR model. We iterate the MRSTAR model and compute:

$$
\xi y_{t:} = \Phi_0 + \Phi_1 y_{t:} + \Phi_2 M_{1:t} + \Phi_3 D E F_{t:} + \Phi_4 P T Y_{t:} + \Phi_5 F_{1}(b_1; b_1; b_2) + \Phi_6 F_{2}(b_2; b_2; b_2)
$$

and

$$
b_t = b + \xi g P_{t:} + \xi k P_{t:} + \xi \gamma_t
$$

where $\gamma_t \sim i i d(0; 3/4)$.

Step 3. Definition of forecasts of the endogenous variables with monetary impulses:

We repeat step 2, but instead of the vector $f r g$, we compute $\xi y_{t+n}$ using a vector $f r g$ with $y_{r t} = \gamma_r I_{t+1}$ and $I_t = 1$ if the monetary shock
occurs at time \( t \) and \( I_t = 0 \) otherwise. \( \gamma_t \) is the \( t^{th} \) observation of the vector of residuals defined in step 1. \( \gamma_t \) is the corresponding standard error. The forecasts obtained are noted \( \xi_y_{r,t+n} \). We do that for \( r = 1; \ldots; R \).

Step 4. Computation of the GIRF

We form the averages

\[
\xi y_{t+n} = \frac{1}{R} \sum_{r=1}^{R} \xi y_{r,t+n} \quad \xi y_{r,t+n}.
\]

This gives us the GIRF of our MRSTAR model, for a given \( t \).

4.3 Application to US data

As explained in the introduction, for nonlinear models the response of the economy varies according to the time period of the shocks. The choice of the initial period of the monetary impulses is thus crucial. We choose the quarters corresponding to the highest and lowest values of \( \beta_2 \) in the MRSTAR equations for 4 GDP and 4 P. Results for the other series are available upon request to authors and are not reported here to avoid an overabundance of figures. The initial periods of the shocks are the following: 1980:4, 1982:3, 1983:1, 1984:4 and 1990:1. We choose different values for \( \gamma_t \) and we distinguish between positive and negative shocks. Given the definition of \( \gamma_t \) in equation (31) and the definition of 4 SL10, a positive monetary shock does correspond to an unexpected decrease of the short term interest rate. Conversely, a negative shock reflects an increase of the short term interest rate. §1; §2 means that we consider §1 and §2 times the residual standard deviation of the shock. Figures 6a-6c and 7a-7c in appendix 3 show the GIRF corresponding to an instantaneous shock. Figures 8a-8c and 9a-9c allow for future shocks: in nonlinear models, the effects of a shock \( \gamma_t \) on 4 GDP_{t+n} or 4 P_{t+n} depend upon the effects of the shocks occurring during the intermediate time periods \( t + 1; \ldots; t + n \).

Several conclusions can be drawn from figures 6a-6c and 7a-7c. The response of the economy to monetary shocks is cyclical. The real GDP fluctuates more than the inflation rate. The observed fluctuations illustrate the fact that the shocks push the economy in different regimes during the propagation time. It is also seen that the shape of the GIRFs are less cyclical when the initial shock is negative than when it is positive. This is in line with the commonly shared view according to which a contractionary monetary policy causes responses that are more persistent.
If we now compare figures 6a-6c to figures 7a-7c, it seems that the response of prices is sluggish as compared to the reaction of the output. The fluctuations are indeed more damped. However, the GIRFs show a larger response of the inflation rate to monetary shocks when those occur before 1990:1 and a smaller response after this date (compare the vertical axes). This would mean that, when there are no regime changes -here after 1990:1 - monetary shocks fail to raise aggregate demand by a lot and prices absorb most of the effects of the shocks. Monetary shocks have very persistent effects on inflation since it is seen that \( P \) is not back to its initial value. Such a conclusion holds particularly when the shock occurs in 1990:1.

To sum up, the GIRFs reproduce some known stylized facts. The MRSTAR models lead to increased output and inflation during the periods following a positive shock. Meanwhile, after some periods, these variables start declining due to regime changes. These changes are due to the feedback effects of our model. An initial shock in equation (1) induces a variation of \( SL \). This affects \( GDP \) and \( P \) through the transition function \( F_2 \). These variations yield a new modification of \( SL \) (equation (3)) and later of \( GDP \) and \( P \). The term structure may vary above or under the threshold value \( b_2 = \frac{5}{5} \), pushing the economy either in depression phases or in expansion regimes.

There are no signs of asymmetry in the sense usually suggested in the literature. For instance, it is often argued that expansionary monetary policies do not affect output, whereas restrictive monetary policies do reduce output. Here, the asymmetric effects are of a different kind: the effects of the shocks vary across time (due to regime shifting phenomena). To stress this, we compute the GIRFs allowing future shocks (see figures 8a-8c and 9a-9c). We see that the profiles of the impulse response functions vary with the initial period of the shocks. Furthermore, it must be noted that the largest output effects come earlier than the largest inflation effects. For instance, when the initial shock occur in 1990:1, the peak of the GIRF correspond to lag 2 for output and to lag 5 for inflation. This contradicts the conclusions found in equilibrium models (see Christiano et alii (1998a, 1998b)). However, these models have been criticized for not allowing enough persistence in the response functions (see, among others, Nelson (1997)). Here, even though the shocks \( \pi_t \) are uncorrelated (since they are residuals of equation (3)), they generate serially correlated responses with long lasting effects. Looking at figures 8a-8c and 9a-9c there is a great evidence of history dependency. With new shocks occurring during the intermediate periods, the endogenous variables are pushed away from their initial value. The shocks can be interpreted as errors made by the central bank when implementing monetary policy. This yields time dependency effects because the policy rule is unsta-
ble. The persistence of the dynamics is a consequence of the nonlinearity -regime shifts- of the MRSTAR equations.

5 Concluding remarks

In this paper, we have proposed a new approach for studying the quantitative effects of monetary policy. The framework of regime-switching models such as the MRSTAR models allow reproducing some stylized facts, notably the persistent responses of inflation and autocorrelated responses in output responses, even when the shocks are independent. Also, the MRSTAR models help reproducing phenomena such as history-dependency, time variability of the impulse functions and sensitivity to the regime observed when the initial shock is produced.

This paper offers several extensions. First, it may be interesting to compare the results obtained here for the US. economy with those of other O.E.C.D. countries. Secondly, the financial transmission channels of monetary policy are often considered without evoking the impact of volatility. Volatility can be a source of instability in the response functions. In this view, it may be worth extending the MRSTAR model by including nonlinear components in the error term. Thirdly, it might be interesting to calibrate and simulate MRSTAR models (instead of estimating them from data) and find the transition function parameters for which the models best reproduce the usual stylized facts on monetary policy.
References


Appendix 1: The data

We use quarterly data over the sample period 1975:1 to 1998:2 for the US economy. The sources are the IMF’s International Financial Statistics and the OECD database. We use of the following variables (seasonally adjusted and transformed in logarithm): M1 for the aggregate quantity of money, the federal fund rate (FED), the treasury bill rate (TBILL), 3-year and 10-year government bond yields (BOND3, BOND10), banks prime loan rate (BANK), the GDP volume, total employment (L), the real labor costs (RCL), hourly earnings (HEARNINGS), the total productivity index (PTY), general government expenditures and investment (I), the output-GAP (GAP), all items consumer prices (P), private final consumption expenditures (CONS). We also consider the US budget deficit (DEF).

The following variables are also defined:

\[ SL_3 = BOND3 - TBILL \] and \[ SL_{10} = BOND10 - TBILL \]
as proxies of interest rate term structures,

\[ CC = FED - BANK \]
as a proxy of the credit channel, and an indicator of the wedge is:

\[ WEDGE = RCL - HEARNINGS + P: \]
Appendix 2: Building and estimating MRSTAR models

Appendix 2.1.- Estimation of the linear models

Note: The p-values corresponding to the parameter estimates or the different test statistics are given in parentheses. F is the Fisher statistic of the LM test of goodness of fit, GB (q) denotes the Godfrey-Breusch statistic of the LM-type test for qth-order serial correlation in the residuals, Skew is the skewness coefficient, Kurt is the Kurtosis, BJ is the Jarque-Bera normality test for the residuals, White is the White heteroskedasticity test, ARCH denotes the Engle conditional heteroskedasticity test and AIC is the Akaike information criteria. $\hat{\theta}^2_{Lin}$ is the estimated variance of the residuals.

Linear model: 4 GDP

$$\begin{align*}
\hat{\theta} \text{ GDP}_t &= 0.0043 + 0.0268 \text{ GDP}_{t-1} + 0.1365 \text{ GDP}_{t-2} \\
&+ 0.0378 \text{ M}_{t-1} + 0.0152 \text{ M}_{t-1} + 0.2193 \text{ M}_{t-2} \\
&+ 0.1552 \text{ M}_{t-3} + 0.7103 \text{ PTY}_t \\
\hat{\theta}^2_{Lin} &= 0.00005, R^2 = 0.300, R^2 = 0.239, F = 4.9082 (0.00), DW = 1.8888, \\
&GB(1) = 1.1769 (0.27), GB(4) = 3.0527 (0.54), Skew = 0.5364, Kurt = 5.0811, BJ = 98.8884 (0.00), White = 17.5883 (0.99), ARCH(1) = 0.1675 (0.68), ARCH(4) = 0.9713 (0.91), AI C = i 9.7181.
\end{align*}$$

Linear model: 4 C

$$\begin{align*}
\hat{\theta} \text{ C}_t &= 0.0019 + 0.0003 \text{ C}_{t-1} + 0.0024 \text{ C}_{t-2} \\
&+ 0.2390 \text{ C}_{t-3} + 0.1938 \text{ M}_{t-1} + 0.0784 \text{ M}_{t-1} \\
&+ 0.3735 \text{ PTY}_t \\
\hat{\theta}^2_{Lin} &= 0.00003, R^2 = 0.307, R^2 = 0.256, F = 5.9913 (0.00), DW = 2.0029, \\
&GB(1) = 0.1535 (0.69), GB(4) = 26854 (0.61), Skew = i 0.9920, Kurt = 3.1133, BJ = 49.9735 (0.00), White = 30.4511 (0.29), ARCH(1) = 0.4631 (0.49), ARCH(4) = 4.4884 (0.34), AI C = i 10.2355.
\end{align*}$$

Linear model: 4 I

$$\begin{align*}
\hat{\theta} \text{ I}_t &= 0.0053 + 0.1139 \text{ I}_{t-1} + 0.0796 \text{ I}_{t-2} \\
&+ 0.2550 \text{ I}_{t-3} + 0.2302 \text{ I}_{t-4} + 0.1914 \text{ PTY}_t \\
&+ 0.3175 \text{ PTY}_{t-1} + 0.0000 \text{ DEF}_t \\
\hat{\theta}^2_{Lin} &= 0.00003, R^2 = 0.307, R^2 = 0.256, F = 5.9913 (0.00), DW = 2.0029, \\
&GB(1) = 0.1535 (0.69), GB(4) = 26854 (0.61), Skew = i 0.9920, Kurt = 3.1133, BJ = 49.9735 (0.00), White = 30.4511 (0.29), ARCH(1) = 0.4631 (0.49), ARCH(4) = 4.4884 (0.34), AI C = i 10.2355.
\end{align*}$$
$b_{Lin}^2 = 0.00009, R^2 = 0.257, \bar{R}^2 = 0.190, F = 3.8104 (0:00), DW = 2.0440,$
$GB (1) = 1.3543 (0:24), GB (4) = 3.6942 (0:44), Skew = 0.0329, Kurt = 0.04574, BJ = 0:0227 (0:98), White = 24:3106 (0:91), ARCH (1) = 1.5922 (0:20), ARCH (4) = 2:0966 (0:71), AIC = j 9:1411.$

**Linear model: 4 L**

\[
\begin{align*}
&c_{L_t} = i 0.0001 + 0.1916c_{L_{1t}} + 0.1168c_{L_{2t}} \\
&i 0.1115c_{L_{3t}} + 0.8275c_{L_{4t}} + 0.2980c_{L_{5t}} \\
&i 0.0212c_{M_{1t}} + 0.0706c_{M_{2t}} + 0.3382c_{PTY_t}
\end{align*}
\]

$\bar{b}_{Lin}^2 = 0.0002, R^2 = 0.900, \bar{R}^2 = 0.890, F = 88.2083 (0:00), DW = 2.0125,$
$GB (1) = 0.4768 (0:48), GB (4) = 2.7285 (0:60), Skew = 0.0449, Kurt = 0.0232, BJ = 0.1885 (0:91), White = 50.2338 (0:24), ARCH (1) = 0.0118 (0:89), ARCH (4) = 2.6111 (0:62), AIC = j 10.6340.$

**Linear model: 4 P**

\[
\begin{align*}
&c_{P_t} = i 0.0006 + 0.6198c_{P_{1t}} + 0.0847c_{P_{2t}} \\
&i 0.3858c_{P_{3t}} + 0.1314c_{M_{1t}} + 0.1390c_{M_{2t}}
\end{align*}
\]

$\bar{b}_{Lin}^2 = 0.0002, R^2 = 0.740, \bar{R}^2 = 0.724, F = 45.5351 (0:00), DW = 1.9810,$
$GB (1) = 0.0240 (0:87), GB (4) = 2.0102 (0:73), Skew = 0.0758, Kurt = 0.0232, BJ = 0.0843 (0:95), White = 28.4175 (0:09), ARCH (1) = 0.2900 (0:59), ARCH (4) = 1.9994 (0:73), AIC = j 10.8410.$

---

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Appendix 2.2.- The different tests

Table A.2.2.1.- Results of the linearity test against STAR alternatives

Note: The number reported are $p$-values: The first column contains the transition variables and the first rows the endogenous variables. We indicate in bold the lowest $p$-values that help selecting the transition variables that will be chosen for $s_1$ in the MRSTAR models.

<table>
<thead>
<tr>
<th>Transition Variables</th>
<th>$\zeta$ GDP</th>
<th>$\zeta$ C</th>
<th>$\zeta$ I</th>
<th>$\zeta$ L</th>
<th>$\zeta$ P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$ SL3t</td>
<td>0.4031</td>
<td>0.3019</td>
<td>0.4245</td>
<td>0.6359</td>
<td>0.9045</td>
</tr>
<tr>
<td>$\zeta$ SL3t 1</td>
<td>0.5287</td>
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<td>0.9550</td>
<td>0.2477</td>
<td>0.2554</td>
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<td>$\zeta$ SL3t 2</td>
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<td>0.2986</td>
<td>0.3217</td>
<td>0.0845</td>
<td>0.2774</td>
</tr>
<tr>
<td>$\zeta$ SL3t 3</td>
<td>0.2525</td>
<td>0.5099</td>
<td>0.5471</td>
<td>0.3989</td>
<td>0.0026</td>
</tr>
<tr>
<td>$\zeta$ SL3t 4</td>
<td>0.2786</td>
<td>0.3049</td>
<td>0.8422</td>
<td>0.0466</td>
<td>0.5413</td>
</tr>
<tr>
<td>$\zeta$ SL10t</td>
<td>0.2242</td>
<td>0.5210</td>
<td>0.6943</td>
<td>0.6157</td>
<td>0.7003</td>
</tr>
<tr>
<td>$\zeta$ SL10t 1</td>
<td>0.0906</td>
<td>0.1113</td>
<td>0.7489</td>
<td>0.3964</td>
<td>0.0565</td>
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<tr>
<td>$\zeta$ SL10t 2</td>
<td>0.0092</td>
<td>0.1985</td>
<td>0.3058</td>
<td>0.2845</td>
<td>0.1573</td>
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<td>$\zeta$ SL10t 3</td>
<td>0.4149</td>
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<td>0.1001</td>
<td>0.4772</td>
<td>0.0531</td>
<td>0.2019</td>
</tr>
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<td>$\zeta$ CCt</td>
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<td>0.3545</td>
<td>0.1492</td>
<td>0.4176</td>
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<td>$\zeta$ CCT 1</td>
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<td>$\zeta$ CCT 2</td>
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<td>$\zeta$ CCT 3</td>
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<td>0.0175</td>
<td>0.0452</td>
<td>0.0937</td>
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<td>$\zeta$ CCT 4</td>
<td>0.7158</td>
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<td>$\zeta$ GAPt</td>
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<td>0.1004</td>
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<td>$\zeta$ GAPt 2</td>
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<td>0.0000</td>
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<td>0.0536</td>
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<td>0.1969</td>
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<tr>
<td>$\zeta$ GAPt 4</td>
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<td>0.0701</td>
<td>0.0839</td>
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</tr>
<tr>
<td>$\zeta$ WEDGEt</td>
<td>0.0266</td>
<td>0.4359</td>
<td>0.7750</td>
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<td>$\zeta$ WEDGET 1</td>
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<td>0.5788</td>
<td>0.0049</td>
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<tr>
<td>$\zeta$ WEDGET 3</td>
<td>0.4927</td>
<td>0.0016</td>
<td>0.3501</td>
<td>0.4342</td>
<td>0.0082</td>
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<tr>
<td>$\zeta$ WEDGET 4</td>
<td>0.1013</td>
<td>0.0615</td>
<td>0.2482</td>
<td>0.7910</td>
<td>0.5363</td>
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</table>
Table A.2.2.2. Choosing between LSTAR and ESTAR models

Note: The number reported are p-values. The first column contains the three nested hypotheses and the first rows the endogenous variables.

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<thead>
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<th></th>
<th>c GDP</th>
<th>c C</th>
<th>c I</th>
<th>c L</th>
<th>c P</th>
</tr>
</thead>
<tbody>
<tr>
<td>H01</td>
<td>0.0392</td>
<td>0.2309</td>
<td>0.0273</td>
<td>0.0017</td>
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<tr>
<td>H02</td>
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<td>0.0031</td>
<td>0.1030</td>
<td>0.1900</td>
<td>0.0002</td>
</tr>
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<td>H03</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.7609</td>
<td>0.5318</td>
<td>0.0662</td>
</tr>
<tr>
<td>LSTR</td>
<td>LSTR</td>
<td>LSTR</td>
<td>LSTR</td>
<td>LSTR</td>
<td>ESTR</td>
</tr>
</tbody>
</table>
Table A.2.2.3. Testing STAR models against MRSTAR alternatives

Note: The number reported are $p$-values. The first column contains the transition variables and the first rows the endogenous variables. We indicate in bold the lowest $p$-values that help selecting the transition variables that will be chosen for $s_2$ in the MRSTAR models.

<table>
<thead>
<tr>
<th></th>
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<th>$\xi \text{ C}$</th>
<th>$\xi \text{ I}$</th>
<th>$\xi \text{ L}$</th>
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<td>0.3957</td>
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<td>0.0198</td>
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Table A.2.2.4. A modified test of STAR models against MRSTAR alternatives

Note: The number reported are $p$ values. The first column contains the transition variables and the first rows the endogenous variables. We indicate in bold the lowest $p$ values that help selecting the transition variables that will be chosen for $s_{\alpha}$ in the MRSTAR models.

<table>
<thead>
<tr>
<th></th>
<th>$\zeta$ GDP</th>
<th>$\zeta$ C</th>
<th>$\zeta$ L</th>
<th>$\zeta$ L</th>
<th>$\zeta$ P</th>
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<td>0.1169</td>
<td>0.1846</td>
<td>-</td>
<td>0.0488</td>
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</tbody>
</table>
Appendix 2.3. - Estimation of the STAR models

Note: The p-values corresponding to the parameter estimates or the different test statistics are given in parentheses. F is the Fisher statistic of the LM test of goodness of fit, GB (q) denotes the Godfrey-Breusch statistic of the LM-type test for qth-order serial correlation in the residuals, Skew is the skewness coefficient, Kurt is the Kurtosis, BJ is the Jarque-Bera normality test for the residuals, White is the White heteroskedasticity test, ARCH denotes the Engle conditional heteroskedasticity test and AIC is the Akaike information criteria. $\hat{\beta}^2_{LSTR}$ is the estimated variance of the residuals.

LSTAR model: 4 GDP

$$
\begin{align*}
\zeta GDP_t & = 0.1518 + 1.3124 \zeta GDP_{t-1} + 3.5300 \zeta GDP_{t-2} \\
& \quad + 0.1725 \zeta M_{t-1} + 0.9483 \zeta M_{t-1} + 2.1429 \zeta M_{t-1} \\
& \quad + 0.9527 \zeta M_{t-3} + 2.5559 \zeta PTY_t \\
& \quad + 0.3150 \zeta M_{t-1} + 2.5516 \zeta GDP_{t-1} + 7.0539 \zeta GDP_{t-2} \\
& \quad + 0.4761 \zeta M_{t-1} + 1.7574 \zeta M_{t-1} + 3.7971 \zeta M_{t-2} \\
& \quad + 1.6293 \zeta M_{t-3} + 5.4875 \zeta PTY_t \\
& \text{£ } 1 + \exp \left(0.0740 \zeta GAP_t + 0.0730 \gamma_{\text{thres}} \right) \\
\end{align*}
$$

$\hat{\beta}^2_{LSTR} = 0.00003$, $\hat{\beta}^2_{LSTR} = 0.6479$, GB (1) = 2.4438 (0.11), GB (4) = 6.6579 (0.15), ET (1) = 4.2982 (0.04), ET (4) = 2.4066 (0.06), Skew = 0.4527, Kurt = 3.066, BJ = 37.4882 (0.00), White = 22.1241 (0.95), ARCH (1) = 0.0613 (0.80), ARCH (4) = 0.1166 (0.99), AIC = 10.1520.
LSTAR model: 4 C

\[ \$ C_t = i \times 0.0062 + 0.2100 \times C_{t-1} + 0.0806 \times C_{t-2} + 0.1216 \times C_{t-3} + 0.1689 \times M_{t} + 0.1850 \times M_{t-1} + 0.0383 \times PTY_t \]

\[ + 0.0145 \times 0.5400 \times C_{t-1} + 0.1099 \times C_{t-2} \]

\[ + 0.1881 \times C_{t-3} + 0.0276 \times M_{t} + 0.2546 \times M_{t-1} + 0.2499 \times PTY_t \]

\[ \mu \] is 0.0062, \( \eta \) is 0.00002, \( \Phi_{LSTR} \) is 0.7198, GB (1) = 0.71 (0.39), GB (4) = 2.8365 (0.58), ET (1) = 0.4424 (0.50), ET (4) = 1.2131 (0.31), Skew = 0.1341, Kurt = 3.8275, BJ = 69.2203 (0.00), White = 37.0812 (0.09), ARCH (1) = 0.7247 (0.39), ARCH (4) = 1.2471 (0.87), AIC = 0.095.

LSTAR model: 4 I

\[ \$ I_t = 0.0157 + 0.0153 \times I_{t-1} + 0.5234 \times I_{t-2} + 0.4388 \times I_{t-3} + 0.0580 \times I_{t-4} + 0.1826 \times PTY_t \]

\[ + 0.0104 \times 0.0157 \times I_{t-1} + 0.8202 \times I_{t-2} \]

\[ + 0.7203 \times I_{t-3} + 0.2686 \times I_{t-4} + 0.2821 \times PTY_t \]

\[ + 0.3089 \times PTY_{t-1} + 0.0002 \times DE \times F_t \]

\[ \mu \] is 0.0157, \( \eta \) is 0.0062, \( \Phi_{LSTR} \) is 0.8828, GB (1) = 0.3448 (0.55), GB (4) = 1.8982 (0.75), ET (1) = 0.0412 (0.83), ET (4) = 0.3175 (0.86), Skew = 0.0588, Kurt = 0.1132, BJ = 0.0919 (0.95), White = 24.5468 (0.90), ARCH (1) = 0.3628 (0.54), ARCH (4) = 3.3600 (0.49), AIC = 0.10623.
LSTAR model: 4 L

\[
\begin{align*}
\mathbf{\zeta}_{L_t} &= 0.0114 + 0.3363\mathbf{\zeta}_{L_{t-1}} + 0.0735\mathbf{\zeta}_{L_{t-2}} \\
&+ 0.5358\mathbf{\zeta}_{L_{t-3}} + 0.6025\mathbf{\zeta}_{L_{t-4}} + 0.5930\mathbf{\zeta}_{L_{t-5}} \\
&+ 0.0311\mathbf{\mu}_{L_t} + 0.0564\mathbf{\mu}_{L_{t-1}} + 0.6119\mathbf{\mu}_{PTY_{t-1}} \\
&+ 0.0232\mathbf{\mu}_{L_{t-1}} + 0.2535\mathbf{\mu}_{L_{t-2}} + 0.1133\mathbf{\mu}_{L_{t-3}} \\
&+ 0.8109\mathbf{\mu}_{L_{t-3}} + 0.4111\mathbf{\mu}_{L_{t-4}} + 0.5539\mathbf{\mu}_{L_{t-5}} \\
&+ 0.0224\mathbf{\mu}_{L_{t-1}} + 0.0955\mathbf{\mu}_{L_{t-1}} + 0.6132\mathbf{\mu}_{PTY_{t-1}} \\
L &= 1 + \exp(-0.2201) \cdot \mathbf{\mu}_{WEDGE_{t-1}} + 0.0155 \cdot \mathbf{\mu}_{3/\mathbf{thres}}
\end{align*}
\]

\[
\begin{align*}
\mathbf{b}^{2}_{LSTR} &= 0.0002, \quad \mathbf{b}^{2}_{STR} = 0.9571, \quad \mathbf{GB} (1) = 2.0229 (0.15), \quad \mathbf{GB} (4) = 4.5477 (0.33), \quad \mathbf{ET} (1) = 0.6122 (0.43), \quad \mathbf{ET} (4) = 1.5842 (0.19), \quad \mathbf{Skew} = 0.0095, \quad \mathbf{Kurt} = 0.1635, \quad \mathbf{BJ} = 0.0971 (0.95), \quad \mathbf{White} = 50.9677 (0.21), \quad \mathbf{ARCH} (1) = 0.2266 (0.63), \quad \mathbf{ARCH} (4) = 2.9223 (0.57), \quad \mathbf{AIC} = 10.6754.
\end{align*}
\]

LSTAR model: 4 P

\[
\begin{align*}
\mathbf{\zeta}_{P_t} &= 0.0051 + 0.4476\mathbf{\zeta}_{P_{t-1}} + 0.0466\mathbf{\zeta}_{P_{t-2}} \\
&+ 0.2276\mathbf{\mu}_{P_{t-3}} + 0.1196\mathbf{\mu}_{L_{t-1}} + 0.1236\mathbf{\mu}_{L_{t-1}} \\
&+ 0.0058 + 0.2001\mathbf{\mu}_{P_{t-1}} + 0.0987\mathbf{\mu}_{P_{t-2}} \\
&+ 0.1664\mathbf{\mu}_{P_{t-3}} + 0.3085\mathbf{\mu}_{L_{t-1}} + 0.0265\mathbf{\mu}_{L_{t-1}} \\
L &= 1 + \exp(-0.9644) \cdot \mathbf{\mu}_{SL10_{t-3}} + 0.1384 \cdot \mathbf{\mu}_{3/\mathbf{thres}}
\end{align*}
\]

\[
\begin{align*}
\mathbf{b}^{2}_{LSTR} &= 0.0002, \quad \mathbf{b}^{2}_{STR} = 0.9152, \quad \mathbf{GB} (1) = 0.1686 (0.68), \quad \mathbf{GB} (4) = 3.3973 (0.49), \quad \mathbf{ET} (1) = 0.0005 (0.98), \quad \mathbf{ET} (4) = 1.0213 (0.40), \quad \mathbf{Skew} = 0.0157, \quad \mathbf{Kurt} = 0.1412, \quad \mathbf{BJ} = 0.0724 (0.96), \quad \mathbf{White} = 27.8473 (0.11), \quad \mathbf{ARCH} (1) = 0.0610 (0.80), \quad \mathbf{ARCH} (4) = 3.5420 (0.47), \quad \mathbf{AIC} = 10.9245.
\end{align*}
\]
Appendix 2.4. - Estimation of MRSTAR models

Note: The p-values corresponding to the parameter estimates or the different test statistics are given in parentheses. \( F \) is the Fisher statistic of the LM test of goodness of fit, \( GB(q) \) denotes the Godfrey-Breusch statistic of the LM-type test for \( q \)-th order serial correlation in the residuals, Skew is the skewness coefficient, Kurt is the Kurtosis, BJ is the Jarque-Bera normality test for the residuals, White is the White heteroskedasticity test, ARCH denotes the Engle conditional heteroskedasticity test and AIC is the Akaike information criteria. \( \hat{\sigma}_{\text{MRSTR}}^2 \) is the estimated variance of the residuals. \( \hat{\sigma}_{\text{thres}}, \hat{\sigma}_{\text{thres1}}^2 \) and \( \hat{\sigma}_{\text{thres2}}^2 \) are the empirical standard deviations of the corresponding transition variables.

**MRSTAR model: 4 GDP**

\[
\begin{align*}
\text{GDP}_t &= \text{M} \cdot 0.0928 + 0.8881 \cdot \text{GDP}_{t-1} + 3.2672 \cdot \text{GDP}_{t-2} \\
&+ 0.1515 \cdot \text{M} \cdot 1.0 + 0.8003 \cdot \text{M} \cdot 1_{t-1} + 1.9143 \cdot \text{M} \cdot 1_{t-2} \\
&+ 1.0683 \cdot \text{M} \cdot 1_{t-3} + 2.4128 \cdot \text{PTY}_t \\
&+ 0.1971 + 2.6647 \cdot \text{GDP}_{t-1} + 6.5502 \cdot \text{GDP}_{t-2} \\
&+ 0.4519 \cdot \text{M} \cdot 1_{t-1} + 1.8207 \cdot \text{M} \cdot 1_{t-2} + 3.4990 \cdot \text{M} \cdot 1_{t-2} \\
&+ 1.7494 \cdot \text{M} \cdot 1_{t-3} + 5.5256 \cdot \text{PTY}_t \\
&+ 0.0497 + 0.2766 \cdot \text{GDP}_{t-1} + 1.2984 \cdot \text{GDP}_{t-2} \\
&+ 0.3045 \cdot \text{M} \cdot 1_{t-1} + 0.4144 \cdot \text{M} \cdot 1_{t-2} + 0.0461 \cdot \text{M} \cdot 1_{t-2} \\
&+ 0.1308 \cdot \text{M} \cdot 1_{t-3} + 0.7176 \cdot \text{PTY}_t \\
&+ 0.1098 + 0.8583 \cdot \text{GDP}_{t-1} + 2.8308 \cdot \text{GDP}_{t-2} \\
&+ 0.5105 \cdot \text{M} \cdot 1_{t-1} + 1.2907 \cdot \text{M} \cdot 1_{t-2} + 0.4340 \cdot \text{M} \cdot 1_{t-2} \\
&+ 0.1309 \cdot \text{M} \cdot 1_{t-3} + 0.8167 \cdot \text{PTY}_t
\end{align*}
\]
with
\[ F_1 = 1 + \exp \left( \mu 0.2471 + \mu 0.5104 \right) \]
\[ F_2 = 1 + \exp \left( \mu 1.7808 + \mu 0.0560 \right) \]

\[ \mu_1 = 0.0003, \mu_2 = 0.57493, GB(1) = 0.06, GB(4) = 4.5070, Skew = 0.1510, Kurt(1) = 1.13428, BJ = 0.06, White = 30.3927, ARCH(1) = 1.1924, ARCH(4) = 2.6955, \]

\[ AIC = 1.02695 \]

**MRSTAR model: 4 C**

\[ \mu C_t = i 0.0063 + 0.1347 C_{t-1} + 0.1389 C_{t-2} + 0.0919 C_{t-3} + 0.2025 M_{1t} + 0.0834 M_{1t-1} + 0.0357 PTY_t \]
\[ + 0.0187 C_{t-1} + 0.5935 C_{t-2} + 0.1052 C_{t-1} + 0.1700 C_{t-3} + 0.0260 M_{1t} + 0.2884 M_{1t-1} + 0.2513 PTY_t F_1 \]
\[ + 0.0152 C_{t-1} + 0.5024 C_{t-2} + 0.3045 C_{t-3} + 0.2771 M_{1t} + 0.0053 M_{1t-1} + 0.1626 PTY_t F_2 \]
\[ + i 0.0276 + 1.3622 C_{t-1} + 0.7042 C_{t-2} + 0.7393 C_{t-3} + 0.3456 M_{1t} + 0.0719 M_{1t-1} + 0.0950 PTY_t F_1 F_2 \]

with
\[ F_1 = 1 + \exp \left( \mu 2.7863 + \mu 0.0691 \right) \]
\[ F_2 = 1 + \exp \left( \mu 2.1912 + \mu 0.0737 \right) \]
\( b_{MRSTR}^2 = 0.00002, \ b_{MRSTR}^{\pm} \ b_{ln}^2 = 0.61792, \ GB(1) = 1.4341 \ (0.23), \ GB(4) = 6.2919 \ (0.17), \ Skew = 1.1328, \ Kurt = 3.6795, \ BJ = 66.9086 \ (0.00), \ White = 35.6764 \ (0.12), \ ARCH(1) = 0.3391 \ (0.56), \ ARCH(4) = 0.9731 \ (0.91), \ AIC = 10.71323.

**MRSTAR model:**

\[
t_i = 0.0217 + 0.6175 t_{lt1} 1 i 0.9024 t_{lt2} 2 \\
+ 0.2589 t_{lt3} 3 i 0.1345 t_{lt4} 4 i 0.1032 P T Y_t \\
+ 0.7689 P T Y_{lt1} 1 + 0.0003 D E F_t \\
+ 0.5653 P T Y_{lt1} 1 + 0.0003 D E F_t 1 F_1 \\
+ 0.0820 + 0.0868 t_{lt1} 1 i 0.1485 t_{lt2} 2 \\
+ 0.6461 t_{lt3} 3 i 1.2715 t_{lt4} 4 i 0.0652 P T Y_t \\
+ 0.5172 P T Y_{lt1} 1 + 0.0023 D E F_t 2 F_2 \\
+ 0.1044 + 0.6454 t_{lt1} 1 i 0.8544 t_{lt2} 2 \\
+ 0.3555 t_{lt3} 3 i 0.7483 t_{lt4} 4 i 0.5838 P T Y_t \\
+ 0.1279 P T Y_{lt1} 1 + 0.0025 D E F_t 2 F_1 \ F_2
\]

with

\[
A F_1 = 1 + \exp i 3.5335 t_{lt3} 3 + 0.0654 /_{thres1} \\
A F_2 = 1 + \exp i 1.8372 t_{lt4} 4 + 0.0778 /_{thres2}
\]

\( b_{MRSTR}^2 = 0.00005, \ b_{MRSTR}^{\pm} \ b_{ln}^2 = 0.5933, \ GB(1) = 0.5668 \ (0.45), \ GB(4) = 1.6293 \ (0.80), \ Skew = 0.0552, \ Kurt = 0.5419, \ BJ = 1.070 \ (0.60), \ White = 31.5513 \ (0.63), \ ARCH(1) = 0.0175 \ (0.89), \ ARCH(4) = 0.8776 \ (0.92), \ AIC = 9.6487.

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MRSTR model: 4 L

$$\begin{align*}
\zeta L_t &= i0.0064 + 0.0977\zeta L_{t-1}i + 0.2427\zeta L_{t-2}i \\
&+ 0.6065\zeta L_{t-3}i + 0.7114\zeta L_{t-4}i + 0.5374\zeta L_{t-5}i \\
&+ i0.0254 M_{1t} + 0.0555\zeta M_{1t-1}i + 0.3254\zeta PTY_t \\
&+ i0.0043 L_{t-1}i + 0.1924\zeta L_{t-2}i \\
&+ 0.5236\zeta L_{t-3}i + 0.1274\zeta L_{t-4}i + 0.7009\zeta L_{t-5}i \\
&+ 0.0098 M_{1t} + 0.0875\zeta M_{1t-1}i + 0.6638\zeta PTY_t F_1 \\
&+ 0.0293 + 0.0229\zeta L_{t-1}i + 0.0404\zeta L_{t-2}i \\
&+ i0.0519 L_{t-3}i + 0.2749\zeta L_{t-4}i + 0.1325\zeta L_{t-5}i \\
&+ 0.1171 M_{1t} + 0.0551\zeta L_{t-1}i + 0.1603\zeta PTY_t F_2 \\
&+ 0.0008 i0.2363\zeta L_{t-1}i + 0.0608\zeta L_{t-2}i \\
&+ i0.1516 L_{t-3}i + 0.0044\zeta L_{t-4}i + 0.1351\zeta L_{t-5}i \\
&+ i0.0746 M_{1t}i + 0.0803\zeta M_{1t-1}i + 0.0986\zeta PTY_t F_1 F_2
\end{align*}$$

with

$$\begin{align*}
F_1 &= 1 + \exp i1.3656\zeta W E D G E_{t-1}i + 0.0035 \zeta \mu \text{thr}_{1} \\
F_2 &= 1 + \exp i1.5084\zeta G A P_{t-2}i + 0.0559 \zeta \mu \text{thr}_{2}
\end{align*}$$

$$\begin{align*}
\beta_{MRSTR}^2 &= 0.00001, \quad \beta_{MRSTR}^2 \pm \beta_{in}^2 = 0.4827, \text{GB (1) } = 0.9650 (0.32), \text{GB (4) } = 10.5763 (0.03), \text{Skew } = 0.5382, \text{Kurt } = 0.3371, BJ = 4.4536 (0.10), \text{White } = 50.1322 (0.24), ARCH (1) = 1.2163 (0.27), ARCH (4) = 5.7042 (0.22), \text{AIC } = i11.3548.
\end{align*}$$
MRSTAR model: 4 P

$c P_t = 0.0011 + 0.3830 c P_{t-1} + 0.0511 c P_{t-2}$
$+ 0.5894 c P_{t-3} + 0.1102 c M_{t-1} + 0.1157 c M_{t-1} M_{t-1}$
$+ i 0.0074 + 0.2001 c P_{t-1} i 0.0982 c P_{t-2}$
$+ 0.2969 c P_{t-3} i 0.3308 c M_{t-1} i 0.0247 c M_{t-1} M_{t-1} F_1$
$+ i 0.0040 i 0.0697 c P_{t-1} + 0.2068 c P_{t-2}$
$+ i 0.4698 c P_{t-3} i 0.2703 c M_{t-1} i 0.0914 c M_{t-1} M_{t-1} F_2$
$+ i 0.0069 + 0.1564 c P_{t-1} i 0.6516 c P_{t-2}$
$+ i 0.4005 c P_{t-3} + 0.3153 c M_{t-1} + 0.2539 c M_{t-1} M_{t-1} F_1 F_2$

with

$N W F_1 = 1 + \exp \frac{\hat{\mu}}{0.9856} + \exp \frac{\hat{\mu}}{0.9903} + \frac{\hat{\mu}}{0.0003} + \frac{\hat{\mu}}{0.49} + \frac{\hat{\mu}}{0.0003} + \frac{\hat{\mu}}{0.49} + \frac{\hat{\mu}}{0.0003} + \frac{\hat{\mu}}{0.49}$

$N W F_2 = 1 + \exp \frac{\hat{\mu}}{0.9327} + \exp \frac{\hat{\mu}}{0.0585} + \frac{\hat{\mu}}{0.0585} + \frac{\hat{\mu}}{0.0585} + \frac{\hat{\mu}}{0.0585} + \frac{\hat{\mu}}{0.0585}$

$b_{MRSTR}^2 = 0.00001, b_{MRSTR}^2 = 0.7973, GB(1) = 3.0759 (0.08), GB(4) = 4.4819 (0.34), Skew = 0.2223, Kurt = 3.7933, BJ = 3.4211 (0.18), White = 16.2177 (0.70), ARCH(1) = 0.7895 (0.37), ARCH(4) = 1.1128 (0.89), AIC = 11.0588.
Appendix 2.5. - Dynamics of MRSTAR models

A.2.5.1. Characteristics roots in each regime for MRSTAR models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regime</th>
<th>Roots</th>
<th>Modulus</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$ GDP</td>
<td>$F_1 = F_2 = 0$</td>
<td>$i \times 0.7056$</td>
<td>0.705</td>
<td>1</td>
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<tr>
<td></td>
<td></td>
<td>$0.43378$</td>
<td>0.433</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 1; F_2 = 0$</td>
<td>$0.270 \times 0.481i$</td>
<td>0.551</td>
<td>3.016</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 0; F_2 = 1$</td>
<td>$i \times 1.067$</td>
<td>1.067</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.475$</td>
<td>0.475</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 1$</td>
<td>$0.183 \times 0.733i$</td>
<td>0.755</td>
<td>3.460</td>
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<table>
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<tr>
<th>Variable</th>
<th>Regime</th>
<th>Roots</th>
<th>Modulus</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$ C</td>
<td>$F_1 = F_2 = 0$</td>
<td>$i \times 1.575 \times 2.037i$</td>
<td>2.575</td>
<td>2.818</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.640$</td>
<td>1.640</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 1; F_2 = 0$</td>
<td>$i \times 1.005 \times 1.009i$</td>
<td>1.424</td>
<td>2.669</td>
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<td></td>
<td></td>
<td>$1.881$</td>
<td>1.881</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 0; F_2 = 1$</td>
<td>$1.159 \times 1.713i$</td>
<td>2.068</td>
<td>6.438</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$i \times 1.099$</td>
<td>1.099</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$F_1 = F_2 = 1$</td>
<td>$i \times 0.229 \times 1.047i$</td>
<td>1.072</td>
<td>3.516</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.249$</td>
<td>1.249</td>
<td>11</td>
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<table>
<thead>
<tr>
<th>Variable</th>
<th>Regime</th>
<th>Roots</th>
<th>Modulus</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$ I</td>
<td>$F_1 = F_2 = 0$</td>
<td>$i \times 1.395 \times 2.504i$</td>
<td>2.867</td>
<td>3.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.432 \times 0.846i$</td>
<td>0.951</td>
<td>5.720</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 1; F_2 = 0$</td>
<td>$i \times 1.108$</td>
<td>1.108</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1.065$</td>
<td>1.065</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$i \times 0.204 \times 1.306i$</td>
<td>1.322</td>
<td>3.640</td>
</tr>
<tr>
<td></td>
<td>$F_1 = 0; F_2 = 1$</td>
<td>$i \times 0.425 \times 0.898i$</td>
<td>0.994</td>
<td>3.120</td>
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<tr>
<td></td>
<td></td>
<td>$0.563 \times 0.634i$</td>
<td>0.848</td>
<td>7.444</td>
</tr>
<tr>
<td></td>
<td>$F_1 = F_2 = 1$</td>
<td>$i \times 0.628 \times 0.619i$</td>
<td>0.882</td>
<td>2.658</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.898 \times 0.169i$</td>
<td>0.914</td>
<td>33.72</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Variable</th>
<th>Regime</th>
<th>Roots</th>
<th>Modulus</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta L )</td>
<td>( F_1 = F_2 = 0 )</td>
<td>( 0.041, 1.027i )</td>
<td>1.028</td>
<td>4:106</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 1.076, 0.878i )</td>
<td>1.389</td>
<td>9:175</td>
</tr>
<tr>
<td></td>
<td>( F_1 = 1; F_2 = 0 )</td>
<td>( 5.163 )</td>
<td>5.163</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 1.142 )</td>
<td>1.142</td>
<td>1</td>
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<tr>
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<td>( 1.108 )</td>
<td>1.108</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 0.033, 0.966i )</td>
<td>0.967</td>
<td>4:090</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>( F_1 = 1; F_2 = 0 )</td>
<td>( 0.936 )</td>
<td>0.936</td>
<td>0:936</td>
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<tr>
<td></td>
<td></td>
<td>( 0.811, 0.821i )</td>
<td>1.154</td>
<td>7:941</td>
</tr>
<tr>
<td></td>
<td>( F_1 = 0; F_2 = 1 )</td>
<td>( 0.992 )</td>
<td>0.992</td>
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<tr>
<td></td>
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<td>( 1.802 )</td>
<td>1.802</td>
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<tr>
<td></td>
<td></td>
<td>( 4.725 )</td>
<td>4.725</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 0.038, 1.065i )</td>
<td>1.065</td>
<td>3:910</td>
</tr>
<tr>
<td>( \zeta P )</td>
<td>( F_1 = F_2 = 0 )</td>
<td>( 0.989 )</td>
<td>0.989</td>
<td>1</td>
</tr>
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<td></td>
<td></td>
<td>( 0.538, 1.193i )</td>
<td>1.309</td>
<td>3:150</td>
</tr>
<tr>
<td></td>
<td>( F_1 = 1; F_2 = 0 )</td>
<td>( 0.847 )</td>
<td>0.847</td>
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<tr>
<td></td>
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<td>( 0.397, 1.083i )</td>
<td>1.153</td>
<td>3:268</td>
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<tr>
<td></td>
<td>( F_1 = 0; F_2 = 1 )</td>
<td>( 1.231 )</td>
<td>1.231</td>
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<tr>
<td></td>
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<td>( 1.693, 1.980i )</td>
<td>2.605</td>
<td>2:757</td>
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<tr>
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<td>( F_1 = F_2 = 1 )</td>
<td>( 29.392 )</td>
<td>29.392</td>
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<tr>
<td></td>
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<td>( 0.675, 1.292i )</td>
<td>1.458</td>
<td>5:770</td>
</tr>
</tbody>
</table>
Appendix 3. Figures

Figure 1a: $\Delta GDP$. Observed values, predictions from linear models and predictions from MRSTAR model

Figure 1b: $\Delta GDP$. First estimated transition function over time (a) and vs first transition variable (b)
Figure 1c: \( \Delta GDP \). Second estimated transition function over time (a) and vs second transition variable (b)

(a)

(b)

Figure 1d: \( \Delta GDP \). Distribution of observations of \( \Delta GDP \) over the different regimes
Figure 2a: $\Delta C$. Observed values, predictions from linear models and predictions from MRSTAR model

Figure 2b: $\Delta C$. First estimated transition function over time (a) and vs first transition variable (b)
Figure 2c: $\Delta C$. Second estimated transition function over time (a) and vs second transition variable (b)

Figure 2d: $\Delta C$. Distribution of observations of $\Delta C$ over the different regimes
Figure 3a: $\Delta l$. Observed values, predictions from linear models and predictions from MRSTAR model.

Figure 3b: $\Delta l$. First estimated transition function over time (a) and vs first transition variable (b).
Figure 3c: $\Delta I$. Second estimated transition function over time (a) and vs second transition variable (b)

Figure 3d: $\Delta I$. Distribution of observations of $\Delta I$ over the different regimes
Figure 4a: $\Delta L$. Observed values, predictions from linear models and predictions from MRSTAR model

Figure 4b: $\Delta L$. First estimated transition function over time (a) and vs first transition variable (b)
Figure 4c: $\Delta L$. Second estimated transition function over time (a) and vs second transition variable (b)

Figure 4d: $\Delta L$. Distribution of observations of $\Delta L$ over the different regimes
Figure 5a: \( \Delta P \). Observed values, predictions from linear models and predictions from MRSTAR model.

Figure 5b: \( \Delta P \). First estimated transition function over time (a) and vs first transition variable (b)
Figure 5c: $\Delta P$. Second estimated transition function over time (a) and vs second transition variable (b)

Figure 5d: $\Delta P$. Distribution of observations of $\Delta P$ over the different regimes
Figure 6a: $\Delta GDP$. Nonlinear impulse response 1980:4

Figure 6b: $\Delta GDP$. Nonlinear impulse response 1982:3
Figure 6c: $\Delta GDP$. Nonlinear impulse response 1990:1

![Graph showing the nonlinear impulse response for GDP with various time horizons and values.]

Figure 7a: $\Delta P$. Nonlinear impulse response 1983:1

![Graph showing the nonlinear impulse response for price change with various time horizons and values.]

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Figure 7b: $\Delta P$. Nonlinear impulse response 1984:4

Figure 7c: $\Delta P$. Nonlinear impulse response 1990:1
Figure 8a: $\Delta GDP$. Nonlinear impulse response 1980:4

Figure 8b: $\Delta GDP$. Nonlinear impulse response 1982:3
Figure 8c: $\Delta GDP$. Nonlinear impulse response 1990:1

Figure 9a: $\Delta P$. Nonlinear impulse response 1983:1
Figure 9b: $\Delta P$. Nonlinear impulse response 1984:4

Figure 9c: $\Delta P$. Nonlinear impulse response 1990:1