Pricing-to-market and limited participation: a joint explanation to the exchange rate disconnect puzzle

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Abstract

Transition to floating exchange rates has led to sharp increases in nominal and real exchange rate volatilities with no corresponding changes in the distribution of macroeconomic fundamentals. This paper investigates the so-called exchange rate disconnect puzzle. In line with the New Open Economy Macroeconomics, Chari, Kehoe and MacGrattan [2000] quantitatively evaluate the role of the interaction between monetary shocks and pricing-to-market (PTM) in exchange rate fluctuations. Even if promising, their results depend on very specific assumptions which cast doubt on the relevance of their explanation. To circumvent this criticism, I investigate the specific channels through which monetary policy affects exchange rates given credit market friction. This paper thus highlights the combined role that PTM and limited participation to the credit market play in the explanation of the exchange rate disconnect puzzle, without relying on the Chari et al.'s specifications. I develop a two-country general equilibrium model based on PTM, credit market rigidities and monetary shocks. Given credit market friction and PTM, monetary shocks generate a nominal exchange rate overshooting which raises the nominal and real exchange rate volatility. The model built on limited participation and PTM improves our understanding of the huge observed exchange rate fluctuations.

Keywords: Exchange rate disconnect puzzle, pricing-to-market, liquidity effect, monetary policy, New Open Economy Macroeconomics

JEL Codes: E32, F41, F31

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1 Introduction

Since the collapse of the Bretton-Woods system in 1973, OECD countries has experienced floating exchange rate regime. One of the main international business cycle features of this period is the extremely volatile behavior of nominal and real exchange rates. This is all the more striking as no corresponding changes appeared in the distribution of macroeconomic fundamentals such as outputs, interest rates and money supplies. This stylized fact is so called the “exchange rate disconnect puzzle” in Obstfeld and Rogoff [2000]’s terminology. These authors thus conclude that

“Exchange rates are remarkably volatile to any model we have of underlying fundamentals (...) and no model seems to be very good at explaining exchange rates even ex-post.” Obstfeld and Rogoff [2000], p.40.

Such a conclusion comes and comforts the pessimistic results of Meese and Rogoff [1983] showing that macroeconomic “models of the Seventies” are unable to beat the naive random walk forecasts of out-of-sample exchange rates. This demonstration thus leads Frankel and Rose [1995] to assert that

“There is remarkably little evidence that macroeconomic variables have consistent strong effects on floating rates”.

However, recent years have witnessed a significant renewal in international macroeconomic theory with the seminal paper of Obstfeld and Rogoff [1995]. The so-called “New Open Economy Macroeconomics” framework gives economists a new insight into the traditional monetary approach of exchange rates first developed by Dornbusch [1976] and Mussa [1986]. The unifying feature of this literature is the introduction of nominal rigidities into a dynamic general equilibrium model based on optimizing agents. This modelling approach sheds light on amplification effects of monetary shocks that may explain the strong disconnect between exchange rates and macroeconomic monetary fundamentals observed in the data.

In line with this literature, I first determine this stylized fact upon which the paper further investigates. For that matter, I compute the volatilities of the nominal exchange rate ($\sigma_e$), the real exchange rate ($\sigma_\Gamma$) and the monetary growth factor ($\sigma_g$) for the six G7 countries vis-à-vis the United States. Results presented in table 1 are derived from quarterly series (OECD sources), on the period 1973:1-1998:4. Series have been previously filtered according to the Hodrick and Prescott [1997]’s method.

Table 1 thus highlights the strong disconnect between exchange rates and monetary growth factor: nominal exchange rate is about 10.37 times more volatile than nominal shocks and real exchange rate almost ten times, for the median of the six G7 countries.
With the development of the “New Open Economy Macroeconomics”, a large number of articles have tried to rationalize the exchange rate disconnect puzzle. They point out the interaction between monetary shocks and price stickiness as the most “popular” story accounting for exchange rates excess volatility. Furthermore, recent research (Betts and Devereux [1996], Sopraseuth [1999]) focusing on the debate regards the choice of currency in which prices are sticky, renews the traditional sticky price approach of exchange rate movements. Indeed, Betts and Devereux [1996] argue that local currency pricing improves the model’s ability to explain exchange rate behavior. The intuition behind this result is straightforward. Pricing-To-Market (PTM, or local currency pricing) implies that some firms are able to set sale prices directly in the buyer’s currency. Hence the law of one price and the purchasing power parity do not necessarily hold, allowing for real exchange rate fluctuations. As pricing-to-market limits the exchange rate pass-through to import prices, it reduces the sensitteness of consumption price indices to nominal exchange rate. As a consequence, any change in the nominal exchange rate following monetary shocks affects the real exchange rate as long as sale prices are sticky. Pricing-to-market thus appears as a key explanation to real exchange rate fluctuations.

Moreover, Betts and Devereux [1996] demonstrate that PTM amplifies the nominal exchange rate response to monetary shocks. Following an expansionary money supply shock, the equilibrium on the money market is ensured by an increase in consumer price indices. However, local currency pricing reduces the impact of nominal exchange rate changes on the domestic and foreign CPI. Hence, given the relative consumption and the relative consumer price levels, the equilibrium on the money market imposes a larger depreciation of domestic currency to compensate for the incomplete exchange rate pass-through. With sticky prices, PTM amplifies the response of both nominal and real exchange rates to monetary shocks. Local currency pricing thus provides a key mechanism that accounts for the high nominal and real exchange rate volatility.

The recent paper by Chari, Kehoe and McGrattan [2000] quantitatively evaluates that story. They thus develop a general equilibrium monetary model with sticky prices and pricing-to-market to investigate the role of monetary shocks in exchange rate fluctuations. They meet some success in reproducing a nominal interest rate decrease (the liquidity effect) and a nominal exchange rate fluctuations.

### Table 1: stylized facts

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_e(%)$</th>
<th>$\sigma_T(%)$</th>
<th>$\sigma_g(%)$</th>
<th>$\bar{\pi}$</th>
<th>$\bar{\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>2.96</td>
<td>3.16</td>
<td>0.82</td>
<td>3.61</td>
<td>3.85</td>
</tr>
<tr>
<td>France</td>
<td>8.48</td>
<td>7.94</td>
<td>0.75</td>
<td>10.71</td>
<td>10.59</td>
</tr>
<tr>
<td>Germany</td>
<td>8.32</td>
<td>8.03</td>
<td>0.90</td>
<td>9.24</td>
<td>9.42</td>
</tr>
<tr>
<td>Italy</td>
<td>8.50</td>
<td>7.84</td>
<td>0.97</td>
<td>8.76</td>
<td>8.08</td>
</tr>
<tr>
<td>Japan</td>
<td>9.34</td>
<td>9.31</td>
<td>0.80</td>
<td>11.68</td>
<td>11.64</td>
</tr>
<tr>
<td>United-Kingdom</td>
<td>8.04</td>
<td>8.00</td>
<td>0.59</td>
<td>13.63</td>
<td>13.56</td>
</tr>
<tr>
<td>Mean</td>
<td>7.61</td>
<td>7.38</td>
<td>0.81</td>
<td>9.61</td>
<td>9.52</td>
</tr>
<tr>
<td>Median</td>
<td>8.40</td>
<td>7.97</td>
<td>0.81</td>
<td>10.37</td>
<td>9.84</td>
</tr>
</tbody>
</table>
overshooting consistent with stylized facts\(^1\), and they obtain theoretical nominal and real exchange rate volatilities that fit the data quite well. However to obtain such results, they need to impose some unappealing assumptions, namely separability between consumption, leisure and money holdings, high risk aversion degree and price stickiness of at least one year. These assumptions cast doubt on the ability of their model to explain exchange rate movements, since a substantial part of their results are likely to derive from the very specific values of the parameters they impose. This leaves the question of the exchange rate excess volatility open: how can we account for the huge disconnect between exchange rates and monetary fundamentals in a more general framework that would not rely on such specifications?

To circumvent this criticism, I investigate on the specific channels through which monetary policy affects exchange rates. This drives me to consider the role of credit market friction in exchange rate volatility. Indeed, if the Chari and al. [2000]'s model is able to generate a liquidity effect and a nominal exchange rate overshooting following monetary shocks, it appears strongly dependent from the somewhat extreme assumptions namely regards utility function. The introduction of the limited participation assumption in the pricing-to-market model may help palliate this limit. For, it could reproduce such a dynamics through very intuitive and simple economic mechanisms while preserving plausible specification choices. The intuition behind this proposal is the following. In line with Dornbusch [1976], the key mechanism behind the overshooting dynamics dwells on the role of the interest rate differential between countries. Given uncovered interest rate parity, following a positive domestic monetary shock the persistent negative interest rate differential has to be offset by an expected appreciation of domestic currency. As expectations are rational, the immediate nominal exchange rate depreciation is followed by an appreciation, that is nominal exchange rate overshooting.

Yet, a generic implication of the standard monetary models -whether they introduce money in the utility function or via a cash-in-advance constraint- is to provoke an increase in the nominal interest rate following a positive monetary shock due to an expected inflation effect. First modelled in a closed economy setting by Christiano [1991], the limited participation assumption makes this kind of models able to reproduce the liquidity effect. It introduces credit market friction through information asymmetries, by assuming that the household chooses the amount of her bank deposits before the occurrence of the monetary injection. As a result, the private amount of loan supply is fixed in the period hence a money supply shock results in a loan supply excess. The loanable funds market equilibrium is therefore ensured by a decrease in the cost of borrowing, that is the nominal interest rate. In a small open-economy setting based on limited participation, Hairault, Patureau and Sopraseuth [2001] show that a positive domestic monetary shock thus

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\(^1\)Indeed, structural VAR models show that a positive monetary shock generates a decrease in the nominal interest rate (the liquidity effect), in a closed economy setting (Christiano and Eichenbaum [1991]). In a multi-country setting, Clarida and Gali [1994], Eichenbaum and Evans [1995], Schlingenhauf and Wrase [1995] demonstrate that a negative US monetary shock implies an increase in the American interest rate and a positive interest rate differential in favor of the United States, as well as a nominal exchange rate over-reaction.
implies a persistent liquidity effect and a subsequent nominal exchange rate overshooting. This particular nominal exchange rate dynamics, that results from rigidities on the market where the interest rate is determined, is shown to substantially account for the huge nominal exchange rate volatility. In a more general framework allowing for deviations from purchasing power parity, monetary shocks are thus likely to generate a substantial disconnect between exchange rates and their monetary fundamentals, since the overshooting dynamics reinforces local currency pricing as amplification mechanism of monetary shocks on nominal and real exchange rates. Introducing the limited participation assumption in the PTM model is likely to raise the magnitude of exchange rate movements. Combining both mechanisms may contribute to bring the theoretical exchange rate volatilities closer to the data.

The other way round, combining local currency pricing and credit market friction may strengthen the occurrence of the liquidity effect as well. Indeed as underlined by Christiano [1991], following a positive money supply shock the liquidity effect may be offset by the expected inflation effect if the monetary shock is persistent enough. As local currency pricing puts a brake on changes in consumption price indices following nominal shocks, it reduces the expected inflation effect and the liquidity effect is more likely to occur. Limited participation and pricing-to-market assumptions can thus be seen as complementary mechanisms since each of them may strengthen the amplification mechanisms of monetary shocks on exchange rates that are specific to the other one. The objective of the paper is to explore these interactions and to investigate the role of monetary shocks in exchange rates fluctuations in this kind of framework.

The paper is organized as follows. Section 2 presents the building blocks of a two-country intertemporal general equilibrium model allowing for monopolistic competition, price stickiness and local currency pricing. In this benchmark model (model M1), I show that PTM and nominal price rigidity are key elements allowing for real exchange rate fluctuations. Yet, the quantitative results are disappointing: a monetary shock implies a monotonic depreciation of domestic currency whose magnitude is not large enough to generate exchange rate volatilities consistent enough with the data. Section 3 then introduces the limited participation assumption in the PTM model. A monetary shock now implies a nominal exchange rate overshooting that stems from the persistent negative interest rate differential. I thus show that the joint assumption of PTM and limited participation enables the model to generate exchange rate volatilities much closer to the stylized facts. Section 4 concludes.

2 Pricing-to-market, monetary policy and exchange rates

2.1 Structure of the model

This section presents the benchmark model based on Betts and Devereux [1998] and Sopraseuth [1999]. The world economy is divided in two countries, country 1 (home) and country 2 (foreign).
Infinitely-lived households in each country consume a continuum of differentiated goods of total measure unity. A proportion $n$ of these goods is produced by the country 1 firms while $(1 - n)$ is produced in country 2.

2.1.1 The household

Preferences of the household are identical between countries. The representative household in each country maximizes

$$E_0 = \sum_{t=0}^{\infty} \beta^t U(C^C_{it}, L_{it})$$

with $C^C_{it}$ the consumption index and $L_{it}$ leisure. As the time endowment is normalized to unity, working time $H_t$ is given by the following equation

$$1 = H_t + L_t$$

The utility function is given by

$$U(C^C_{it}, L_{it}) = \ln C^C_{it} + \gamma H \ln(L_{it})$$

As agent $i$ consumes goods from both countries, her bundle of goods is defined by

$$C^C_{it} = \left[ \int_0^1 c_{it}(z) \frac{dz}{\eta} \right]^{\frac{1}{\eta}}$$

where $\eta > 1$ represents the elasticity of substitution of goods and $c_{it}(z)$ the quantity of good $z$ consumed by agent $i$. We assume that all goods intervene a symmetric way in the consumption bundle. Goods produced by country 1 monopolistic firms are indexed by $z \in [0, n]$ and goods produced by country 2 monopolistic firms are indexed by $z \in [n, 1]$.

As in Blanchard and Kiyotaki [1987] the country $i$ household’s optimal allocation between goods leads to

$$c_{it}(z) = \left[ \frac{\varsigma_t(z)}{P^C_{it}} \right]^{-\eta} C^C_{it}$$

with $\varsigma_t(z)$ the price of good $z$ and $P^C_{it}$ the country $i$ consumption price index. Both prices are expressed in country $i$ currency. In both countries all firms can “price-to-market” (PTM) by setting different prices for the local and the foreign markets directly in the buyer’s currency. Hence the law of one price does not necessarily holds in the world economy. Good $z$ sold in country $i$ is $p_{it}(z)$ if firm $z$ is local (from country $i$), or $p^*_j(z)$ if the firm is from country $j$ ($j \neq i$). Both prices are set in the country $i$ currency.
The consumption-based indexes $P_{it}^C$, $i = 1, 2$ based on the above preferences are

$$P_{1t}^C = \left[ \int_0^n p_{1t}(z)^{1-\eta} dz + \int_0^n p_{2t}(z)^{1-\eta} dz \right]^{1/\eta}$$  \hspace{1cm} (3)$$

$$P_{2t}^C = \left[ \int_0^n p_{1t}(z) dz + \int_0^n p_{2t}(z)^{1-\eta} dz \right]^{1/\eta}$$  \hspace{1cm} (4)

Because all firms directly set prices in the currency of the buyer, the nominal exchange rate does not enter the domestic and foreign price indexes.

**Program of the representative household** Each period the country $i$ household chooses her amount of consumption index $C_{it}^C$, her leisure time $L_{it}$ and the amount of money holdings needed to consume because of the cash-in-advance constraint she faces:

$$P_{it}^C C_{it}^C \leq M_{it}$$  \hspace{1cm} (5)

Moreover the household can also save by holding foreign assets. Financial markets are complete and for each state of nature $s$ there is a contingent claim $B_i(s_{t+1})$ bought by country $i$ household at period $t$ that yields one unit of domestic currency if at period $t+1$ the realized state is $s_{t+1}$.

I present the country 1 household’s program. The foreign household’s problem can be inferred by symmetry. The domestic household maximizes equation (1) subject to the cash-in-advance constraint (5) and the following budget constraint

$$P_{it}^C C_{it}^C + M_{it} + \int \chi(s_{t+1}) B_1(s_{t+1}) ds_{t+1} \leq M_{it} + P_{it} w_{it} H_{it} + B_1(s_t) + \int_0^n \pi_{1t}(z) dz$$  \hspace{1cm} (6)

The household enters the period with the predetermined amounts $M_{it}$ and $B_1(s_t)$. $w_{it}$ denotes the real wage and $\int_0^n \pi_{1t}(z) dz$ the profits of the domestic firms which are returned to the household (the owner of the domestic firms) at the end of the period.

The optimization program of the domestic household is written as a Bellman equation

$$V[M_{1t}, B_1(s_t)] = \max_{\{C_{it}^C, L_{it}, M_{it+1}, B_{it+1}\}} \left\{ U(C_{it}^C, L_{it}) + \beta \int V[M_{it+1}, B_1(s_{t+1})] f(s_{t+1}, s_t) \right\}$$

subject to equations (5) and (6). $\theta_1$ and $\lambda_1$ are the multipliers respectively associated with each constraint. The function $f(s_{t+1}, s_t)$ is the density function that describes how $s_t$ becomes $s_{t+1}$.

The first order conditions are then

$$U''_{C_{it}^C} = P_{1t}^C (\theta_{1t} + \lambda_{1t})$$  \hspace{1cm} (7)$$

$$-U''_{H_{it}} = P_{it} w_{it} \lambda_{1t}$$  \hspace{1cm} (8)$$

$$\chi(s_{t+1}) = \beta \lambda_{1t+1} / \lambda_{1t} f(s_{t+1})$$  \hspace{1cm} (9)$$

$$\lambda_{1t} = \beta E_t \left[ U_{C_{it+1}^C} / P_{it+1}^C \right]$$  \hspace{1cm} (10)
After deriving the first order conditions for the foreign household, we obtain strict equality between the budget multipliers (based on the optimal choices of contingent claims)

\[ \lambda_{1t} = \lambda_{2t}, \forall \ t \]

In real terms it becomes:

\[ \Lambda_{2t} = \Gamma_t \Lambda_{1t} \]  

with \( \Gamma_t = \frac{e_t P^C_t}{P^C_t} \) the real exchange rate, \( \Lambda_{1t} \) and \( \Lambda_{2t} \) the marginal wealth in each country.

2.1.2 The firms

The structure of the goods market relies on Blanchard and Kiyotaki [1987]. Technologies are identical across countries and across firms. Each firm sets her prices and quantities taking indexes prices and aggregate demand functions as given. The country \( i \) firm \( z \) accumulates physical capital \( k_{it}(z) \), determines her labor demand \( h_{it}(z) \), the prices \( p_{it}(z), p^*_it(z) \) and quantities \( x_{it}(z), x^*_it(z) \) sold on markets \( i \) and \( j (j \neq i) \) respectively.

The production technology is given by a Cobb-Douglas function:

\[ F_{it}(k_{it}(z), h_{it}(z)) = k_{it}(z)^\alpha h_{it}(z)^{1-\alpha} \]

Total individual output is sold on both markets implying that

\[ x_{it}(z) + x^*_it(z) = k_{it}(z)^\alpha h_{it}(z)^{1-\alpha} \]

The firm \( z \) physical stock evolves according to

\[ k_{it+1}(z) = (1 - \delta)k_{it}(z) + I^c_{it}(z) \]

with \( I^c_{it}(z) \) the investment bundle of firm \( z \) and \( \delta \) the depreciation rate For sake of simplicity I assume that the investment index has the same structure than the consumption one:

\[ I^c_{it}(z) = \left[ \int_0^1 i_{it}(z,l) \frac{e_l}{P^C_t} \, dl \right]^{\frac{\eta}{1-\eta}} \]

This implies that the optimal investment demand for good \( l \) by firm \( z \) (in country \( i \)) is

\[ i_{it}(z,l) = \left[ \frac{\varsigma_{it}(l)}{P^C_t} \right]^{-\eta} I^c_{it}(z) \]

where \( \varsigma_{it}(l) \) stands for \( p_{it}(l) \) if the firm selling goods \( l \) is from country \( i \) and \( p^*_j(l) \) if it belongs to country \( j \). I assume that each firm faces adjustment costs on capital payed in terms of composite

\[ As I focus attention on the consequences of monetary shocks, I abstract from technological process in the production function as a simplifying assumption. \]
price adjustment costs are given by

\[ CI^C_{it}(z) = \frac{\phi}{2}(k_{it+1}(z) - k_{it}(z))^2 = \left[ \int_0^1 c_{it}(z,l) \frac{dl}{l} \right]^{\eta} \]

Moreover, I introduce nominal price rigidity in line with the “menu costs” literature. I thus assume that the firms face quadratic costs for each price \( p_{it} \) and \( p_{i1}^t \). As in Hairault and Portier [1995], price adjustment costs are given by

\[ CP_{it}^C = \frac{\Phi}{2} \left( \frac{p_{it}(z)}{p_{t-1}(z)} - \pi \right)^2 \]
\[ CP_{it}^{\pi C} = \frac{\Phi}{2} \left( \frac{p_{i1}^t(z)}{p_{t-1}^i(z)} - \pi \right)^2 \]

As \( \pi \) represents the steady state price growth rate, adjustment costs are null at the long term equilibrium. These costs are paid in terms of composite goods:

\[ CP_{it}^C(z) = \left[ \int_0^1 c_{it}(z,l) \frac{dl}{l} \right]^{\eta} \quad \text{with} \quad c_{it}(z,l) = \left[ \frac{s_{it}(l)}{P_{it}^C} \right]^{\eta} \quad CP_{it}^C \]
\[ CP_{it}^{\pi C}(z) = \left[ \int_0^1 c_{it}(z,l) \frac{dl}{l} \right]^{\eta} \quad \text{with} \quad c_{it}(z,l) = \left[ \frac{s_{i1}(l)}{P_{it}^{\pi C}} \right]^{\eta} \quad CP_{it}^{\pi C} \]

The country \( i \) firm \( z \) faces two demand functions from the local market \( x_{it}^d(z) \) and the foreign one \( x_{it}^{x d}(z) \). For a country 1 firm these demand functions are

\[ x_{1t}^d(z) = \left[ \frac{p_{1t}(z)}{P_{1t}^C} \right]^{\eta} D_{1t}^C \]
\[ x_{it}^{x d}(z) = \left[ \frac{p_{i1}^t(z)}{P_{i1}^{x t}} \right]^{\eta} D_{2t}^C \]

with \( D_{1t}^C = C_{1t}^C + I_{1t}^C + C1_{1t}^C + CP_{it}^C + CP_{it}^{\pi C} \) total demand in country \( i, i = 1, 2 \).

The program of the country 1 firm that maximizes its intertemporal profit is written as a Bellman equation:

\[ V[p_{1t-1}(z), p_{i1t-1}^t(z), k_{1t}(z)] = \max \left\{ p_{1t}(z)x_{1t}(z) + c_{1t}i_{1t}(z)x_{1t}^*(z) - P_{1t}^C w_{1t}h_{1t}(z) - P_{1t}^C I_{1t}^C(z) - P_{1t}^{\pi C} \left( \frac{p_{i1}(z)}{P_{i1}^C} - \pi \right)^2 - P_{1t}^{\pi C} \left( \frac{p_{i1}^t(z)}{P_{i1}^{\pi t}} - \pi \right)^2 \right\} \]

subject to the sequence of constraints

\[ x_{1t}(z) + x_{1t}^*(z) = k_{1t}(z) \eta h_{1t}(z)^{1-\alpha} \]
\[ x_{1t}(z) \leq \left[ \frac{p_{1t}(z)}{P_{1t}^C} \right]^{\eta} \quad D_{1t}^C \quad \text{and} \quad \left( n_{1t}(z) \right) \]
\[ x_{1t}^*(z) \leq \left[ \frac{p_{i1}^t(z)}{P_{i1}^{x t}} \right]^{\eta} \quad D_{2t}^C \quad \text{and} \quad \left( n_{1t}^{x t}(z) \right) \]
\[ k_{1t+1}(z) = (1-\delta)k_{1t}(z) + I_{1t}^C(z) \]
The first order conditions are given by the following equations:

\[
p_{1t}(z) - \nu_{1t}(z) = e_t p_{1t}(z) - \nu_{1t}(z) \\
P_{1t} w_{1t} = \frac{p_{1t}(z)}{1 + \mu_{1t}(z)} \left[ (1 - \alpha) x_{1t}(z) + x_{1t}(z) \right] \\
P_{1t} q_{1t}(z) = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_{1t}} \left[ \frac{p_{t+1}(z)}{1 + \mu_{t+1}(z)} \left[ \alpha \frac{x_{t+1}(z) + x_{t+1}(z)}{k_{t+1}(z)} \right] \right] \right\} \\
\]

\[
x_{1t}(z) + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_{1t}} P_{1t} \Phi \frac{p_{t+1}(z)}{p_{1t}(z)} \left( \frac{p_{t+1}(z)}{p_{1t}(z)} - \pi \right) \right\} \\
= P_{1t} \Phi \frac{p_{1t}(z)}{p_{1t-1}(z)} \left\{ \frac{p_{1t}(z)}{p_{1t-1}(z)} - \pi \right\} + \eta \nu_{1t}(z) x_{1t}(z) \\
x_{1t}^{**}(z) + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_{1t}} P_{1t} \Phi \frac{p_{t+1}^{**}(z)}{p_{1t}^{**}(z)} \left( \frac{p_{t+1}^{**}(z)}{p_{1t}^{**}(z)} - \pi \right) \right\} \\
= P_{1t} \Phi \frac{p_{1t}^{**}(z)}{p_{1t-1}(z)} \left\{ \frac{p_{1t}^{**}(z)}{p_{1t-1}(z)} - \pi \right\} + \eta \nu_{1t}(z) x_{1t}^{**}(z) \\
\]

Because of the market segmentation, the firm sets its prices \( p_{1t}(z) \) and \( p_{1t}^{**}(z) \) according to the localization of the buyer, both prices being linked through equation (18). Equations (19) and (20) express the demand for labor and capital respectively. Equation (19) shows that the usual equality between real wage and labor marginal productivity does not hold. Monopolistically competing firms set prices such that labor marginal productivity is above real wage: Their market power allows them to pay the workforce below her marginal productivity thus to extract positive rents. As well the expected marginal return of an investment in physical capital is given by the expected resell price of capital \( \left( \frac{p_{1t+1}(z)}{1 + \mu_{1t+1}(z)} \left[ \alpha \frac{x_{t+1}(z) + x_{t+1}(z)}{k_{t+1}(z)} \right] \right) \) which is lower than the true marginal productivity \( (p_{1t+1}(z) \alpha \frac{x_{t+1}(z) + x_{t+1}(z)}{k_{t+1}(z)}) \) (see appendix A). Each single firm is enticed to hire and invest less in the monopolistic competition set-up than in a perfect competition framework.

Price decisions (equations(21) and (22)) show that, absent nominal price rigidity (\( \Phi = 0 \)), the mark-up is constant and equal to

\[
\mu = \frac{1}{\eta - 1}
\]

On the contrary when firms face adjustment costs on prices (\( \Phi > 0 \)) the mark-up are endogenous and fluctuate following monetary innovations.

### 2.1.3 The central bank

Each period and in each country, the representative household receives money transfers from the monetary authorities. The monetary aggregate evolves as

\[
M_{it+1} = g_{it} M_{it}
\]

The monetary growth factor \( g_{it} \) evolves according to an autoregressive stochastic process:

\[
\begin{pmatrix}
\log g_{1t+1} \\
\log g_{2t+1}
\end{pmatrix} = \begin{pmatrix} \rho_{g1} & 0 \\ 0 & \rho_{g2} \end{pmatrix} \begin{pmatrix}
\log g_{1t} \\
\log g_{2t}
\end{pmatrix} + \begin{pmatrix} 1 - \rho_{g1} & 0 \\ 0 & 1 - \rho_{g2} \end{pmatrix} \begin{pmatrix}
\log g_{1} \\
\log g_{2}
\end{pmatrix}
\]
where \( \log g_t \) is the mean of the process and \( \{ \varepsilon^g \}_t \) is the vector of monetary innovations serially independent and correlated between countries. We have \( E[\varepsilon^g_1] = E[\varepsilon^g_2] = 0 \) and \( E[\varepsilon^g_1, \varepsilon^g_2] = 0 \). The parameter \( \psi_g \) governs the cross-correlation between monetary innovations.

### 2.1.4 Equilibrium

I consider symmetric equilibria where all firms in the same country set the same prices, that is \( p_{it}(z) = p_{it}^* \) and \( p_{it}^*(z) = p_{it}^*, \forall \ i = 1, 2 \). Consequently they face identical demand functions and decisions regarding production, labor and capital are similar. Symmetry thus simplifies the expression of the price indexes

\[
P_{1t}^C = \left[ np_{1t}^{1-\eta} + (1 - n)p_{2t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (23)
\]

\[
P_{2t}^C = \left[ np_{2t}^{1-\eta} + (1 - n)p_{1t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (24)
\]

The different markets are equilibrated:

- **Labor market**

  \[
  H_{1t} = nh_{1t} \quad (25)
  \]
  \[
  H_{2t} = (1 - n)h_{2t} \quad (26)
  \]

- **Physical capital market**

  \[
  K_{1t}^C = nk_{1t} \quad (27)
  \]
  \[
  K_{2t}^C = (1 - n)k_{2t} \quad (28)
  \]

- **Financial markets**

  \[
  B_1(s_t) + B_2(s_t) = 0, \quad \forall s_t \quad (29)
  \]

Finally the world budget constraint stems from the households’ budget constraints, the governments’ budget constraints, the market equilibrium conditions and the expressions of the firms profits:

\[
D_{1t}^C + \Gamma_t D_{2t}^C = n \left( \frac{p_{1t}x_{1t} + \varepsilon_{1t}p_{1t}^*x_{1t}^*}{P_{1t}^C} \right) + (1 - n)s\Gamma_t \left( \frac{p_{2t}x_{2t} + \varepsilon_{2t}p_{2t}^*x_{2t}^*}{P_{2t}^C} \right) \quad (30)
\]

After the stationarizing of equations, I determine the long-run equilibrium. The equations are then log-linearized around the steady state according to the Farmer [1993]’s methodology.
2.2 Calibration

The period in the model is assumed to be a quarter. The calibration of the parameters is presented in table 2. $\gamma_H$ is chosen such as working time is 20%. As I consider similar countries I choose $n = 0.5$. The discount factor $\beta$ is 0.988 which corresponds to a 1.2% steady state quarterly interest rate. Since 1945 the wage to GDP ratio is around 0.58 in the United States which implies $\alpha = 0.42$. The depreciation of capital is about 10% a year or $\delta = 0.025$ per quarter. To calibrate the elasticity of substitution between goods I use the steady state relation between $\mu$ and $\eta$: $\mu = \frac{1}{\eta-1}$, referring on Morrison [1990]'s estimation $\mu = 0.197$. To calibrate the parameter $\Phi$ of the adjustment costs on prices I proceed the following way: I calculate the unconditional absolute volatility of consumer price index for the United States on the 1973:1-2000:2 period and I thus determine the value for $\Phi^3$ consistent with this estimation that is $\Phi = 10$. I adopt the same reasoning to calibrate the parameter $\phi$ that scales adjustment costs on physical capital. After estimating the unconditional volatility of investment relative to output on the same data set$^4$, I derive the correct theoretical value for $\phi$ that is equal to 0.5. The calibration of the monetary process is based on Kollmann [1999], who gives an estimate of the joint process followed by the monetary aggregates of the United States and its G7 partners between 1973:3 and 1994:4. The cross-correlation of the monetary innovations is 0.20 that gives the value for $\psi_g^5$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\Phi$</th>
<th>$\gamma$</th>
<th>$\mu$</th>
<th>$n$</th>
<th>$\rho_a$</th>
<th>$\sigma_{\varepsilon g}$</th>
<th>$\psi_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.42</td>
<td>0.988</td>
<td>0.025</td>
<td>10</td>
<td>0.5</td>
<td>0.197</td>
<td>0.5</td>
<td>0.3</td>
<td>0.009</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2: Parameters

2.3 Monetary shocks, dynamic path and volatilities of exchange rates

2.3.1 Domestic monetary policy and impulse response functions

I now focus on the effects of monetary policy by first deriving the impulse response functions following a 1% increase in the domestic money growth factor. Figure 1 presents the transition

\[
\begin{pmatrix}
\log a_{1t+1} \\
\log a_{2t+1}
\end{pmatrix} = \begin{pmatrix}
\rho_a & \rho_{a12} \\
\rho_{a12} & \rho_a
\end{pmatrix} \begin{pmatrix}
\log a_{1t} \\
\log a_{2t}
\end{pmatrix} + \begin{pmatrix}
1-\rho_a & 0 \\
0 & 1-\rho_a
\end{pmatrix} \begin{pmatrix}
\log a \\
\log a
\end{pmatrix} \\
+ \begin{pmatrix}
1 & \psi_a \\
\psi_a & 1
\end{pmatrix} \begin{pmatrix}
\varepsilon_{1t+1} \\
\varepsilon_{2t+1}
\end{pmatrix}
\]

with $\rho_a = 0.906, \rho_{a12} = 0.088, \psi_a = 0.13$ and $\varepsilon^a = 0.00852$.

$^4$I use the OCDE database for the 1973:1-2000:2 period. The unconditional absolute consumption price index volatility equates 1.50%. To correctly reproduce this volatility in the theoretical set-up I add a technological stochastic process identified by Backus, Kehoe and Kydland [1995] as

\[
\begin{pmatrix}
\log a_{1t+1} \\
\log a_{2t+1}
\end{pmatrix} = \begin{pmatrix}
\rho_a & \rho_{a12} \\
\rho_{a12} & \rho_a
\end{pmatrix} \begin{pmatrix}
\log a_{1t} \\
\log a_{2t}
\end{pmatrix} + \begin{pmatrix}
1-\rho_a & 0 \\
0 & 1-\rho_a
\end{pmatrix} \begin{pmatrix}
\log a \\
\log a
\end{pmatrix} \\
+ \begin{pmatrix}
1 & \psi_a \\
\psi_a & 1
\end{pmatrix} \begin{pmatrix}
\varepsilon_{1t+1} \\
\varepsilon_{2t+1}
\end{pmatrix}
\]

$^5$The empirical estimation of $\sigma_I/\sigma_Y$ is 2.75% for the United States.

Indeed both values are linked through the following equation:

\[
\rho(\varepsilon^g_{1t}, \varepsilon^g_{2t}) = \frac{2\psi_g}{1 + \psi_g^2}
\]
dynamics for nominal interest rates and the nominal exchange rate\(^6\).

Figure 1: Return differential and nominal exchange rate dynamics

With the relative increase in domestic money supply, the loss of purchasing power for domestic currency implies an immediate nominal exchange rate depreciation. As the domestic monetary shock raises current and expected inflation, the opportunity cost of money holdings, that is the nominal interest rate, increases on impact because of the expected inflation effect in country 1\(^7\). In foreign country, the nominal interest rate slightly increases since inflation weakly reacts to the domestic monetary shock. As a result the interest rate differential remains positive until both nominal interest rates have come back to their steady state values.

This is of particular interest regards the nominal exchange rate dynamics. Perfect capital mobility implies that uncovered interest rate parity holds. This imposes any return differential to be offset by expected changes in nominal exchange rate. Since the responses of nominal interest rates are determined by the Fisherian fundamentals, the persistent positive interest rate differential requires an expected depreciation of domestic currency. As expectations are rational, nominal exchange rate goes on depreciating after its immediate increase. Figure 1 displays the continuous depreciation of the nominal exchange rate along its transition path that is crucially linked to the

\(^6\)For a detailed analysis of the real effects of monetary shocks with nominal price rigidity and pricing-to-market, see Betts and Devereux [1998] and Sopraseuth [1999]. Impulse response functions for the whole set of variables is displayed in appendix B.

\(^7\)The nominal interest rate is given by the following definition:

\[
\frac{1}{1 + R_1} = \beta E_t \left[ \frac{\lambda_{1t+1}}{\lambda_{1t}} \right], \quad \text{and} \quad \frac{1}{1 + R_2} = \beta E_t \left[ \frac{\lambda_{2t+1}}{\lambda_{2t}} \right]
\]

or

\[
\frac{1}{1 + R_1} = \beta E_t \left[ \frac{\Lambda_{1t+1}/\Pi_{1t+1}}{\Lambda_{1t}/\Pi_{1t}} \right], \quad \text{and} \quad \frac{1}{1 + R_2} = \beta E_t \left[ \frac{\Lambda_{2t+1}/\Pi_{2t+1}}{\Lambda_{2t}/\Pi_{2t}} \right]
\]
persistent increase in the interest rate differential \((R_1 - R_2)\).

Figure 1 shows that the 1% increase in the domestic money growth factor results in a almost similar 1% increase in the nominal exchange rate. This suggests that the nominal exchange rate dynamics only reflects the monetary shock: the model lacks amplification mechanisms and the quantitative effects of the monetary shocks on exchange rates are likely to be limited.

Figure 2: Monetary shock and real exchange rate response

Figure 2 presents the dynamic path for real exchange rate following a 1% increase in the domestic money growth factor. To understand this impulse response function let’s recall that the real exchange rate dynamics combines the nominal exchange rate response and the relative price index movements (in log):

\[
\hat{\Gamma}_t = \hat{\epsilon}_t + \hat{P}_{2t} - \hat{P}_{1t}
\]

Nominal price rigidity induces firms to react demand changes by adjusting mark-ups rates and quantities rather than sale prices, therefore putting a brake on the consumer price index changes following monetary shock. Price stickiness is a key transmission channel of nominal exchange rate movements to real exchange rate. Furthermore, recent international business cycle models argue that PTM strengthens this channel by limiting the exchange rate pass-through to import prices (Betts and Devereux [1996], Betts and Devereux [1998], Sopraseuth [1999]). A generic implication of the domestic monetary shock is the immediate nominal exchange rate depreciation. If firms set prices in their home currency (no PTM, producer currency pricing) the import prices decrease in country 2 exerts a down pressure on the foreign price level. On the contrary in country 1 the increase in export prices raises the domestic CPI. When all firms set prices in the buyer’s currency, nominal exchange rate does not enter both index price indices. As a result, following monetary shocks, consumption price indexes will change only with sale prices variations, whose magnitude is limited because of nominal rigidities. Pricing-to-market and price stickiness are thus key elements for any movement in nominal exchange rate to affect real exchange rate.
Besides, Betts and Devereux [1996] argue that PTM amplifies the nominal exchange rate response to monetary shocks. Following an expansionary money supply shock, the equilibrium on the money market is usually ensured by an increase in consumer price indices. However, pricing-to-market prevents this adjustment mechanism from occurring. To compensate for the incomplete exchange rate pass-through, the money market equilibrium imposes a larger depreciation of domestic currency. Yet, from a quantitative point of view figure 1 shows that monetary policy has a limited impact on the nominal exchange rate depreciation. This failure is due to the combination of the lack of liquidity effect and the uncovered interest rate parity. Indeed, the lasting positive interest rate differential imposes a nominal exchange rate monotonic depreciation given uncovered interest rate parity. The model fails to generate any substantial disconnect between the nominal exchange rate and its monetary fundamentals. The weak magnitude of nominal exchange rate variations is likely to imply limited real exchange rate fluctuations despite pricing-to-market and price stickiness. Next section quantitatively evaluates this intuition.

2.3.2 Monetary innovations and volatility

This section analyzes the ability of the PTM model to account for the huge observed exchange rates fluctuations. The statistics reported in table 3 are obtained from 500 simulations of the theoretical series that are filtered according to the Hodrick and Prescott [1997]'s methodology. I report here the exchange rates cyclical properties for a wide range of values of the parameter $\Phi$.

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\sigma_\varepsilon$ ($%$)</th>
<th>$\sigma_\Gamma / \sigma_g$ (%)</th>
<th>$\sigma_\varepsilon / \sigma_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.59</td>
<td>1.69</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>1.56</td>
<td>1.64</td>
<td>0.11</td>
</tr>
<tr>
<td>10</td>
<td>1.60</td>
<td>1.70</td>
<td>0.50</td>
</tr>
<tr>
<td>25</td>
<td>1.59</td>
<td>1.68</td>
<td>0.73</td>
</tr>
<tr>
<td>50</td>
<td>1.57</td>
<td>1.65</td>
<td>0.91</td>
</tr>
<tr>
<td>Data</td>
<td>8.40</td>
<td>10.37</td>
<td>7.97</td>
</tr>
</tbody>
</table>

Table 3: Exchange rates volatilities in the PTM model

Table 3 presents the quantitative results that confirm the previous reasoning (section 2.3.1). When $\Phi = 0$ prices are perfectly flexible, the law of one price and the purchasing power parity hold; consequently the real exchange rate is constant equal to one. This is no more the case when prices are sticky ($\Phi > 0$) and directly set in the buyer’s currency. Because of the deviations from purchasing power parity it implies, local currency pricing plays a key role in any convincing monetary explanation of real exchange rate fluctuations. Nevertheless the amplification mechanisms in the model are insufficient enough to generate any disconnect between the nominal exchange rate and its monetary fundamentals. As a result the theoretical exchange rate volatilities remain far lower than the empirical ones.

To better reproduce the empirical exchange rates cyclical properties I now introduce the limited participation assumption in the pricing-to-market model. In line with the Hairault and al. [2001]'s
work, the objective is to now generate an over-reaction of nominal exchange rate with monetary
impulses. Given nominal rigidity and PTM this new mechanism is likely to enlarge the real
exchange rate fluctuations as well and get the model closer to the stylized facts identified in table
1.

3 Pricing-to-market behavior, limited participation and monetary policy: the role on exchange rate volatility

3.1 Timing of the period

The world economy still consists of two countries, 1 and 2 respectively sized $n$ and $1 - n$. Each
country is now lived by four types of agents: a representative household, monopolistic good-
producing firms, a government-central bank and perfectly competing financial intermediaries. The
timing of the decisions within a period can be separated in five steps:

- At the beginning of the period the monetary shock occurs: in both countries the monetary
  authorities inject liquidity into the loanable funds markets.

- In each country the credit market opens and firms determine their demand for loans. As in
  Dow [1995] they borrow cash from financial intermediaries to finance investment in physical
capital. Note that the current supply of loans is already known given the households’ deposit
inherited from previous period decisions and the central bank liquidity injection. The firms
also determine their demand for labor and capital.

- In the third step transactions on the labor market and the goods market occur. Each firm
  sells its production to the local and foreign households after setting goods sale prices directly
  in the buyer’s currency, given aggregate demand in each country.

- In each country the representative household determines her contingent claims portfolio.
  Labor income is collected and loans are repaid to the financial intermediary for a net nominal
  interest rate $R_{it}$. The household receives interest payments on her deposits as well as dividend
  payments from the local banks and the local firms (as owner of both).

- At the end of the period the households choose the amount of bank-deposit $M_{it}^b$ to put
  in the banks the next period and the amount $M_{it}^c$ of money-cash allowing consumption
  purchases the next period.

3.2 Structure of the model

3.2.1 The household

Only the intertemporal household’s problem is now changed by the introduction of the limited
participation assumption. As in Andolfatto and Gomme [2000], I consider that in the current
period, the country $i$ household chooses the amount of deposits she wants to put into the bank
the next period. Thus at the end of period $t$ she decides the amount $M_{ct}^{t+1}$ of money available for consumption purchases (money-cash) in period $t + 1$ and the amount $M_{dt}^{t+1}$ of money to put into the bank (money-deposit).

Furthermore, following Christiano and Eichenbaum [1992] and King and Watson [1996] in a closed economy framework, I introduce adjustment costs on money holdings. Indeed, if the standard limited participation model (Christiano [1991]) generates a liquidity effect following a positive monetary shock, the decrease in the interest rate is not strong and persistent enough as compared to the stylized facts. As modeled by Christiano and Eichenbaum [1992], one way to improve the liquidity effect is to modify the environment so that the financial sector remains more “liquid” than the real sector for several periods after the shocks. Hairault and al. [2001] show the relevance of this assumption in a small-open economy framework. Since adjustment costs on money holdings imply a more negative interest rate differential, they amplify the nominal exchange rate overshooting dynamics. They thus contribute to disconnect nominal exchange rate movements from monetary fundamentals. I model the Christiano and Eichenbaum [1992]’s intuition by assuming that adjusting money-cash $M_{ct}^{t}$ is costly. In a stochastic environment subject to monetary shocks, changing the consumption scheme or equivalently money holdings, is time-costly: reorganizing the flow of funds deprives the household from time available to leisure or working activities. As a consequence the time endowment (normalized to unity) is now written as:

$$1 = L_{it} + H_{it} + \Omega_{it}$$

where $\Omega_{it}$ is the cost of changing money holdings that I suppose to be quadratic:

$$\Omega_{it} = \frac{\xi}{2} \left( \frac{M_{ct}^{t+1}}{M_{ct}^{t}} - g \right)^2$$

$g$ represents the money holdings growth rate at the steady state level.

Each period the representative household in country $i$ chooses her amount of aggregate consumption and her labor supply. Savings can take the form of money holdings, bank deposits or contingent claims. The optimization program for the domestic household is written as a Bellman equation:

$$V \left[ M_{ct}^{t}, M_{dt}^{t}, B_{1}(s_{t}) \right] = \text{Max} \left\{ U(C_{ct}^{t}, L_{it}) + \beta \int V \left[ M_{ct+1}^{t}, M_{dt+1}^{t}, B_{1}(s_{t+1}) \right] f(s_{t+1}, s_{t}) \right\}$$

subject to

$$P_{ct}^{C} C_{ct}^{t} \leq M_{ct}^{t} \quad (\theta_{1t})$$

$$P_{ct}^{C} C_{ct+1}^{t} + M_{ct+1}^{b} + \int \chi(s_{t+1})B_{1}(s_{t+1})ds_{t+1} \leq M_{ct}^{c} + P_{it} w_{it} (1 - L_{it} - \Omega_{it}) + (1 + R_{it})M_{it}^{b} + B_{1}(s_{t}) + \int_{0}^{n} \pi_{1t}(z)dz + \pi_{1t}^{b} \quad (\lambda_{1t})$$

17
where $\pi^b_{it}$ represents the profits received from the banks. The first-order conditions are:

\[
U'_{C_{it}} = P_t^C (\theta_{it} + \lambda_{it}) \tag{31}
\]
\[
U'_{L_{it}} = P_t^C w_{it} \lambda_{it} \tag{32}
\]
\[
\lambda(s_{it+1}) = \beta \lambda_{it+1} f(s_{it+1}) \tag{33}
\]
\[
\lambda_{it} = \beta E_t [(1 + R_{it+1}) \lambda_{it+1}] \tag{34}
\]

\[
\lambda_{it} + P_t^C w_{it} \lambda_{it} \left( \frac{1}{M_{it}} \left( \frac{M_{it+1}}{M_{it}} - g \right) \right) = \beta E_t \left[ \frac{U'_{C_{i+1}}}{P_{t+1}} \right] \tag{35}
\]

\[
+ \beta E_t \left[ \frac{P_{t+1} w_{it+1} \lambda_{it+1}}{M_{it+1}} \left( \frac{M_{it+2}}{M_{it+1}} - g \right) \right]
\]

The program of the country 2 household is can be inferred by symmetry. The optimal choices of contingent claims portfolio still imply strict equality between the budget multipliers ($\lambda_1 = \lambda_2$). The optimality conditions (31), (32), (33) are similar the previous ones (section 2.1.1). Equation (34) represents the optimal choice of bank deposits: the marginal cost of increasing deposits ($\lambda_{it}$) equates the expected marginal wealth it yields tomorrow given the future interest rate. Equation (35) represents the optimal choice of money holdings. Let’s note that it corresponds to equation (10) for $\xi = 0$. The left member of the equation represents the marginal cost of one extra money-cash unit. The marginal cost does not only consists in the budget cost $\lambda_{it}$ since the household has now to give up available time represented here in terms of wage loss ($\frac{\partial \lambda_{it+1}}{\partial M_{it+1}} > 0$). The right member of equation (35) represents the marginal benefit of increasing $M_{it+1}$: the amount of money-cash asked today for tomorrow entices the household to purchase goods the next period given the future purchasing power of money, and it now implies saving time tomorrow ($\frac{\partial \lambda_{it+1}}{\partial M_{it+1}} < 0$). On optimum the marginal cost and the marginal benefit of increasing money holdings are equal.

From the optimal choices of bank deposits in each country and given that $\lambda_1 = \lambda_2$, I derive the uncovered interest rate parity equation (expressed in log):

\[
\frac{1}{1 + R} E_t \tilde{R}_{it+1} - \frac{1}{1 + R} E_t \tilde{R}_{it+1} = E_t \tilde{e}_{it+1} - \tilde{e}_t \tag{36}
\]

### 3.2.2 The firms

The firms program is only slightly modified. Each firm now takes into account the borrowing cost of investment. This implies the following Bellman equation

\[
V [p_{it-1}(z), p'_{it-1}(z), k_{it}(z)] = Max \left\{ \begin{array}{l}
\Phi_1 [p_{it}(z) x_{it}(z) + \epsilon_t p_{it}(z) x_{it}(z) - P_t^C w_{it} h_{it}(z) - (1 + R_{it}) P_{it}^C I_{it}(z)] \\
- P_1^C \left( \frac{P_{it}^C}{P_{it-1}(z)} - \frac{P_t^C}{P_{it-1}(z)} \right)^2 - P_t^C \left( \frac{P_{it}^C}{P_{it-1}(z)} - \frac{P_t^C}{P_{it-1}(z)} \right)^2 \\
- P_{it}^C \left( k_{it+1}(z) - k_{it}(z) \right)^2 + \int \chi(s_{it+1}) V [p_{it}(z), p'_{it}(z), k_{it+1}(z)] \end{array} \right\}
\]

subject to the same constraints i.e. equations (14), (15), (16), (17).
Only the first order condition for investment decision has changed to become

\[ P^{C}_t(q_t(z)) + R_t = \beta E_t \left\{ \frac{\lambda_{1t+1}}{\lambda_{1t}} \left[ \frac{\rho_{1t+1}(z)}{\rho_{1t}(z)} + \frac{\sigma_{1t+1}(z)}{\sigma_{1t}(z)} \right] + P^{C}_{t+1}(q_{t+1}(z) - \delta + (1 - \delta)R_{1t+1}) \right\} \]  
(37)

The nominal interest rate now intervenes in the arbitrage condition. The marginal cost of investment is equal to the shadow price of capital \( q_t \) plus the borrowing cost \( R_t \). The expected marginal return consists in the marginal product to be sold tomorrow given the mark-up, net from depreciation, plus what is saved from not buying this extra unit of capital tomorrow, that is the expected nominal interest rate and the future shadow price of capital.

### 3.2.3 The central bank

The program of monetary authorities is identical to the one described in section 2.1.3.

### 3.2.4 The financial intermediaries

In each country financial intermediaries perfectly compete with each other. They accept bank deposits from the households \( (M^b_{it+1}) \) that are paid back at the end of the period for a nominal interest rate \( R_t \). They also receive cash injection \( X_{it} \) from their local central bank. Banks resources are loaned to local firms that shall borrow to invest in physical capital. The end-of-period profits are given back to the owner (the household) as dividends. The asset balance of the representative bank leads to

\[ D_{it} = M^b_{it} + X_{it}, \quad \forall i = 1, 2 \]

where \( D_{it} \) represents the loans granted to the firms \( i \). At the end of the period the bank dividends are

\[ \pi^b_{it} = (1 + R_{it})D_{it} - (1 + R_{it})M^b_{it}, \quad \forall i = 1, 2 \]  
(38)

Combining both previous equations gives the expression for the profit of the bank:

\[ \pi^b_{it} = (1 + R_{it})X_{it}, \quad \forall i = 1, 2 \]

### 3.2.5 Equilibrium

At the symmetric equilibrium, consumer price indexes are defined through equations (3) and (4). The different markets (labor, capital, goods, contingent claims) are equilibrated according to equations (25) to (29). The loanable funds market equilibrium in each country is given by:

\[ P^{C}_t I^{C}_t = M^b_{it} + (g_{it} - 1)M_{it}, \quad \forall i = 1, 2 \]  
(39)

The world budget constraint is described by equation (30).
3.3 Calibration

The set of parameters \( \{\alpha, \beta, \delta, \mu, n, \rho_g, \sigma_g, \psi_g\} \) is set identical to the previous one (section 2.2). At the long-run equilibrium, the steady-state money holdings ratio \( g \) equates long-run inflation \( \pi \). To calibrate the price adjustment cost parameter \( \Phi \) and the capital adjustment cost parameter \( \phi \) in this new theoretical environment I adopt the same strategy than previously. I set \( \Phi \) and \( \phi \) so as to mimic the cyclical behavior of the consumption price index and investment for the United States on the period 1973-2000. I thus obtain \( \Phi = 9 \) and \( \phi = 0.95 \). The remaining free parameter is the degree \( \xi \) of adjustment costs on money holdings. In line with Christiano and Eichenbaum [1992] and King and Watson [1996], I seek to analyze the consequences of introducing very small adjustment costs on money holdings. Absent any reference value for this parameter in the literature, the relevance of the chosen values for \( \xi \) is assessed regards the corresponding cost of steady state leisure and in terms of minute per week, according to the Hairault and al. [2001]’s method. For the benchmark calibration, the chosen value \( \xi = 1 \) means that a 1% increase in the ratio \( \frac{M_{it+1}}{M_{it}} \) beyond its stationary value costs 0.03% of the steady state leisure. When I translate the amount of time lost in rearranging portfolio into minutes per week, I obtain that it represents a cost of around 0.2 minute per week\(^8\): I still consider very small adjustment costs.

3.4 Effects of monetary policy shocks

3.4.1 Domestic monetary policy and impulse response functions

Figure 3 presents the impulse response functions of the nominal interest rates and the nominal exchange rate with the domestic monetary expansionary policy.

Despite the immediate increase in domestic inflation, the domestic interest rate now decreases on impact to further come back to its initial steady state level. A monetary expansion thus implies a persistent liquidity effect which is consistent with econometric studies (Clarida and Gali [1994], Eichenbaum and Evans [1995], Christiano and Eichenbaum [1992]). In the foreign country inflation slightly increases while the nominal interest rate slightly contracts. As the liquidity effect is stronger in the home country, the interest rate differential \( (R_1 - R_2) \) remains negative for several periods after the shock. Given uncovered interest rate parity, the nominal exchange rate has to depreciate beyond its new steady state level for the agents to expect a further appreciation of domestic currency. As a result the nominal exchange rate dynamics exhibits over-reaction. Given uncovered interest rate parity and rational expectations, the specific path of the nominal exchange rate stems from the persistent negative interest rate differential: the mechanisms on the loanable funds market where the nominal interest rates are determined, are key elements to understand such impulse response functions.

\(^8\)Juster and Stafford [1991] estimate available time not allocated to work for the United States in 1981. Our calculations are based on their results. The mean for both men and women says that 70 hours per 168 h (a week) are personal time.
Domestic loanable funds market

The total supply of loans increases with the monetary injection since the household’s deposits are predetermined. All things ever equal the nominal interest rate -the cost of borrowing- should decrease for the firms accept to absorb the loan supply excess. Yet, the immediate nominal interest rate response is non trivial as loan demand also increases with the monetary injection. First the positive wealth effect induced by the monetary injection affects the auctioneers’s valuation of a marginal investment and entices firms to invest more. Second, as firms are reluctant to adjust sale prices, the positive demand shock implies a persistent contraction in the mark-up rates. As shown in equation (37) this raises the marginal return of investment. The stickier sale prices are, the more loan demand increases for a given interest rate and offsets the initial loan supply excess. In the first period the liquidity effect prevails only for limited variations of loan demand, that crucially depends upon the nominal price rigidity degree.

As shown by Christiano and Eichenbaum [1992] and King and Watson [1996] the persistence of the liquidity effect is closely related to the private loan supply behavior. In the first period the household chooses the amount of money-cash and money-deposit for the next period. Because of the expected inflation effect, the household is willing to increase her money holdings to preserve her future consumption. However it is costly for the household to raise the ratio $\frac{M_{ct+1}}{X_{ct}}$ dramatically since it deprives her for time available to working activities or leisure. According to equation (35), larger adjustment costs makes more expensive any variation in the money holdings ratio in the first period and the household will rather wait. As a result in the period of the monetary injection the household increases $M_{ct+1}$ only by a small amount. She rather prefers increasing her bank deposits, taking moreover into account that the domestic wealth effect is maximum in the period of
the monetary injection. The second period on, the significant increase in private deposits enlarges the effects of the monetary shock and explains the persistent reduction in the domestic interest rate.

The behavior of loan supply plays a crucial role in explaining the magnitude and the persistence of the liquidity effect the periods after the shock. The relative contraction in loan demand acts the same way. With the immediate boom in investment, capital marginal productivity decreases the second period on. Furthermore the mark-up rates gradually come back to their initial steady state level as the effects of the monetary shock vanish. Both reasons entice firms to disinvest. The second period on, loan demand contracts while loan supply goes on increasing. The combination of both effects explains the large and persistent liquidity effect.

**Foreign country loanable funds market**

In the period of the domestic monetary injection, loan supply does not change. The nominal interest rate decrease stems from loan demand variations. Whereas country 1 aggregate demand increases with the domestic shock, foreign aggregate demand slightly contracts because of a negative wealth effect. Indeed, in real terms the budget multipliers are linked by the following equation (in log):

$$\tilde{\Lambda}_{2t} = \tilde{\Lambda}_{1t} + \tilde{\Gamma}_t$$

The period of the shock the real exchange rate depreciation counteracts the positive domestic wealth effect and the foreign country suffers a weak wealth contraction, and aggregate demand slightly reduces. This exerts a down pressure on foreign firms investment. The stickier sale prices are, the more loan demand contracts and the loanable funds equilibrium is ensured by a decrease in the nominal interest rate.

Figure 4: *Monetary shock, liquidity effect and real exchange rate*

![Figure 4: Monetary shock, liquidity effect and real exchange rate](image)

Figure 4 displays that the real exchange rate monotonically depreciates following a 1% increase.
in the domestic money growth shock. If the dynamic path of real exchange rate hardly differs from the one displayed in figure 2, the magnitude of the immediate depreciation is more than twice higher than in the PTM model. This comes from the higher nominal exchange rate depreciation on impact. Given nominal price rigidity, pricing-to-market puts a brake on domestic inflation and brings the foreign price level up. Thus the limited participation assumption enables the model to generate a nominal exchange rate overshooting, and following monetary shocks, the real exchange rate response is also magnified through pricing-to-market. Theoretical exchange rates volatilities are thus likely to get closer to the data. Beyond the qualitative response of exchange rates to monetary shocks, I now want to assess whether the model can substantially account for the huge observed exchange rate fluctuations.

3.4.2 Monetary innovations and volatility

In this section I focus on the quantitative effects of the introduction of the limited participation assumption in the pricing-to-market model. Does the nominal exchange rate overshooting dynamics play a role in nominal and exchange rate volatility? To answer that question, I compare the cyclical properties of exchange rates in both models given identical parameters values. Table 4 thus presents the statistical results for both sets of parameters (sections 2.2 and 3.3). The statistics are obtained from 500 simulations of the series that are filtered according through the Hodrick and Prescott [1997]'s method.

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_e (%) )</th>
<th>( \frac{\sigma_e}{\sigma_g} )</th>
<th>( \sigma_r (%) )</th>
<th>( \frac{\sigma_r}{\sigma_g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi = 0.5, \Phi = 10 )</td>
<td>M1</td>
<td>1.608</td>
<td>1.696</td>
<td>0.498</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>2.227</td>
<td>2.349</td>
<td>1.137</td>
</tr>
<tr>
<td>Calibration 2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi = 0.95, \Phi = 9 )</td>
<td>M1</td>
<td>1.599</td>
<td>1.687</td>
<td>0.473</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>2.507</td>
<td>2.645</td>
<td>1.343</td>
</tr>
</tbody>
</table>

Table 4: Quantitative results of both models

Whatever the parameters values, nominal and real exchange rates volatilities are higher in the M2 model. Following monetary shocks, rigidities on the credit market imply an overshooting of the nominal exchange rate that significantly affects its volatility. Because of pricing-to-market, the real exchange rate inherits the large nominal exchange rate fluctuations. As a result both nominal and real exchange rates volatilities get closer to the stylized facts. The limited participation assumption given pricing-to-market helps disconnecting exchange rates movements from those of the macroeconomic fundamentals, namely the monetary growth factor. Both mechanisms thus improve our understanding of the huge observed exchange rate fluctuations.

The analysis conducted in section 3.4.1 suggests that two parameters play a key role in such results: the degree of nominal price rigidity \( \Phi \) and the parameter of the adjustment costs on money holdings \( \xi \). I thus proceed to a sensitivity analysis to both parameters.
Role of credit market rigidity  Figure 5 presents the nominal exchange rate dynamics for three values of $\xi = 0, 5, 10$.

Figure 5: Credit market rigidities and nominal exchange rate overshooting

As adjustment costs on money holdings increase, the magnitude of the nominal exchange rate overshooting gains in magnitude and persistence. This comes from the impulse response function of the interest rate differential. High adjustment costs on money holdings enlarge the domestic liquidity effect. As the gap between foreign and domestic returns widens, uncovered interest rate parity implies agents to expect a strong domestic currency appreciation. On impact the nominal exchange rate has to increase far beyond its new steady state level. Note that the simple limited participation model ($\xi = 0$) is unable to generate any nominal exchange rate overshooting: the nominal exchange rate dynamics is similar to the one displayed in model 1. This result relies on the lack of persistence of the liquidity effect. As equation (36) shows, the key element for a domestic monetary impulsion in period $t$ to generate a nominal exchange rate overshooting is the negative expected interest rate differential in period $t + 1$. Hence monetary policy should entail a persistent liquidity effect. Christiano [1991] and Christiano and Eichenbaum [1992] in a closed economy setting show that the simple limited participation model hardly replicates a liquidity effect that persists beyond the first period. The impulse response functions displayed here confirm these results. Without adjustment costs on money holdings, the interest rate differential turns out to be positive in favor of the domestic country the second period on. Given uncovered interest rate parity the nominal exchange rate displays a monotonic depreciation towards its new steady state level. This result highlights the crucial role played by rigidities on the credit market.

Figure 6 displays the impulse response functions of the real exchange rate for the chosen values of $\xi$. The immediate real exchange rate depreciation is strengthened by the degree of money holdings adjustment costs. As the price index ratio ($P^*_2 / P^*_1$) is not significantly influenced by rigidities on
the credit market, the larger real exchange rate increase mainly derives from the nominal exchange rate response. As shown in figure 5 credit market rigidities substantially enlarges the magnitude of the nominal exchange rate overshooting. Since pricing-to-market limits consumption price index ratio changes following monetary shocks, credit market rigidities also enlarge the magnitude of the real exchange rate depreciation.

The next step is to quantitatively evaluate the role of the credit market rigidities. Table 5 presents the results of the sensitivity analysis for five values of $\xi$. The statistics are obtained from 500 simulations of the series that have been filtered according to the Hodrick and Prescott [1997]'s method.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\sigma_e$ (%)</th>
<th>$\sigma_g$ (%)</th>
<th>$\sigma_{\Gamma}$ (%)</th>
<th>$\sigma_{\Gamma g}$ (%)</th>
<th>min/week</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.665</td>
<td>1.756</td>
<td>0.60</td>
<td>0.632</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2.507</td>
<td>2.645</td>
<td>1.343</td>
<td>1.417</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>3.265</td>
<td>3.443</td>
<td>1.554</td>
<td>1.640</td>
<td>1.3</td>
</tr>
<tr>
<td>10</td>
<td>3.845</td>
<td>4.055</td>
<td>1.713</td>
<td>1.806</td>
<td>2.5</td>
</tr>
<tr>
<td>15</td>
<td>4.277</td>
<td>4.511</td>
<td>1.819</td>
<td>1.918</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5: The role of credit market friction

Rigidities on the credit market substantially improve the predictions of the model concerning the volatility of nominal and real exchange rate. Indeed, the theoretical nominal exchange rate can be four times more volatile than monetary shocks, and the real exchange rate almost twice. In line with the Hairault and al. [2001]'s work, the results presented in table 5 confirm that overshooting significantly affects nominal exchange rate volatility. Pricing-to-market then acts as an key transmission channel of nominal exchange rate movements to real exchange rate. Nominal and real exchange rates fluctuations are disconnected from fundamentals. If the simple limited participation model (with $\xi = 0$) is unable to generate plausible exchange rates fluctuations, only
small adjustment costs are needed to substantially improve the theoretical results. Yet, only small adjustment costs added to frictions in the credit market are needed to generate plausible exchange rates fluctuations following monetary shocks. In accordance with Dornbusch [1976] and Hairault and al. [2001], following monetary impulses overshooting plays a key role in explaining the high volatility of the nominal exchange rate. Beyond the results of both papers, I show that overshooting also plays a key role in explaining the real exchange rate volatility.

**Role of nominal price rigidity** I now analyze the role played by nominal price rigidities by conducting a sensitivity analysis to the parameter $\Phi$. Figures 7 and 8 respectively present the impulse response functions of nominal and real exchange rates to a 1% increase in the domestic money growth factor, for three values of $\Phi$.

Figure 7: Nominal price rigidities, monetary shock and nominal exchange rate

![Figure 7](image)

Section 3.4.1 made clear that price stickiness counteracts the liquidity effect in country 1 by provoking an immediate increase in loan demand. As a result it reduces the gap between domestic and foreign returns following a domestic monetary expansion. Given uncovered interest rate parity agents expect a limited appreciation of domestic currency. Figure 7 confirms that a higher degree of nominal price rigidity reduces the magnitude on the overshooting of nominal exchange rate.

The real exchange rate dynamics stems from both elements: the movements of the nominal exchange rate ($e$) and the consumption price indexes ratio ($P^C_2/P^C_1$). On one hand a high degree of price stickiness limits the immediate increase in the nominal exchange rate hence reduces the immediate real exchange rate depreciation. On the other hand, nominal price rigidity and pricing-to-market prevent consumption price indices from excessive fluctuations, since price rigidity limits changes in sale prices and pricing-to-market rule out the exchange rate pass through. This effect amplifies real exchange rate movements following monetary impulses. Figure 8 makes clear that the
relative price effect dominates since a higher degree of price rigidity raises the magnitude of the real exchange rate depreciation. Even if they reduce the immediate response of the nominal exchange rate, nominal price rigidity and PTM are key propagation mechanisms of monetary impulses to real exchange rates by theirs effects on consumption price indices.

The next step is to check the role of $\Phi$ on the exchange rates cyclical properties. Table 6 presents the quantitative results in terms of exchange rates volatilities. The statistics are obtained from 500 simulations of the series that are filtered according through the Hodrick and Prescott [1997]'s method.

Table 6: The role of nominal price rigidity

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\sigma_e(%)$</th>
<th>$\frac{\sigma_e}{\sigma_\mu}$</th>
<th>$\sigma_\Gamma$</th>
<th>$\frac{\sigma_\Gamma}{\sigma_e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.40</td>
<td>2.52</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>2.36</td>
<td>2.49</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>2.32</td>
<td>2.44</td>
<td>1.18</td>
<td>1.24</td>
</tr>
<tr>
<td>15</td>
<td>2.24</td>
<td>2.36</td>
<td>1.32</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Table 6 confirms that nominal price rigidities substantially increase the real exchange rate volatility, as they reduce the nominal exchange rate fluctuations simultaneously. Because price stickiness limits the disconnect between the nominal exchange rate and its monetary fundamentals, the nominal exchange rate volatility is reduced. Nominal rigidities exert a complex influence on exchange rates. They simultaneously prevent monetary innovations to generate dramatic nominal exchange rate fluctuations whereas they allow nominal exchange rate changes to influence the real exchange rate movements by limiting price indices to react monetary shocks.

The sensitivity analysis to $\Phi$ confirms the generic result that nominal price rigidity substantially contribute to the real exchange rate fluctuations. However the striking result here consists in the down pressure price stickiness exerts on the nominal exchange rate volatility. Nevertheless,
from a quantitative point of view, the results presented in table 4 show that the model based on limited participation and pricing-to-market generates a disconnect between exchange rates and their monetary fundamentals that improves the theoretical cyclical properties of nominal and real exchange rates.

4 Conclusion

The extremely volatile exchange rates behavior since the collapse of the Bretton-Woods system is a well-known stylized fact that questions economists since the seminal Meese and Rogoff [1983]’s paper. In line with the New Open Economy Macroeconomics, recent international business cycle literature stresses out the role of the interaction between monetary shocks, sticky prices and pricing-to-market in the huge exchange rates movements. As shown by Betts and Devereux [1996], pricing-to-market limits the exchange rate pass-through to import prices. It therefore reduces the sensitiveness of consumption price indices to monetary shocks provided for price stickiness. PTM thus amplifies the response of both nominal and real exchange rates to monetary shocks. The Chari and al. [2000]’s paper attempts to quantitatively evaluate this story in an intertemporal general equilibrium model. They obtain nominal and real exchange rates volatilities that fit the data quite well. However to obtain such results they impose some unappealing assumptions, namely high risk aversion degree and price stickiness of at least one year that limits the explanatory power of the pricing-to-market model regards exchange rate movements.

To palliate the limits of the Chari and al. [2000]’s modelling, I investigate the joint role of local currency pricing and credit market friction in explaining the exchange disconnect puzzle. Some intuitive reasoning indeed suggests that combining both mechanisms may improve the results. I thus develop an intertemporal general equilibrium model based on sticky prices, local currency pricing and limited participation. If PTM is a key ingredient for real exchange rate fluctuations, simulation results are quantitatively disappointing. Following a positive domestic monetary shock, the persistent positive interest rate differential implies a monotonic depreciation of domestic currency given uncovered interest rate parity. The magnitude of the implied nominal exchange rate dynamics is not large enough to entail exchange rate volatilities consistent with the data. The introduction of the limited participation model substantially improves the results. A positive domestic monetary shock now generates a persistent liquidity effect that translates into a persistent negative interest rate differential and a nominal exchange rate overshooting given uncovered interest rate parity. The overshooting dynamics substantially raises the nominal and real exchange rate volatility. The model based on both limited participation and PTM therefore explains a substantial part of the huge disconnect between exchange rates and their monetary fundamentals. Credit market rigidities and local currency pricing are complementary mechanisms that play a key role in understanding the extremely volatile behavior of exchange rates observed in the data.
References


A Monopolistic competition, nominal rigidity and mark-up rates

Since firms compete monopolistically in the world economy, their market power entices them to break the usual optimality condition of equality between good sale price and unit cost. Each individual price-maker firm $z$ (from country $i$) sets its price by adding a mark-up rate to its unit cost. Mark-up rate is thus defined as

$$p_{it}(z) = (1 + \mu_{it}(z))Cm_{it}(z)$$

where $p_{it}(z)$ stands for the goods price set by the firm $z$ and $Cm_{it}(z)$ is the marginal cost of production:

$$Cm_{it}(z) = \frac{\partial CT_{it}(z)}{\partial F_{it}(z)}$$

The analytical expression of the mark-up first requires to derive the total cost of production for the firm $z$. This is not obvious since I assume that the firms invest in physical capital. I thus proceed in two steps. Consider that each firm actually consists in two distinct units. The first one, called the production unit, chooses its optimal amounts of capital and labor so as to minimize production costs given the production technology. She rents the required capital units to the second firm unit for a real rental rate $z_{it}$. $w_{it}$ is the real wage. The second division, the strategy department, sets the optimal couple {price, production} to maximize the discounted stream of dividends payments, given the aggregate demand functions it faces.

- **First step : Optimal demand for production factors**

  The objective is to minimize the total cost of production for a given amount of production:

  $$\min_{\{h_{it}(z),k_{it}(z)\}} CT_{it}(z) = P^C_{it}w_{it}h_{it}(z) + P^C_{it}z_{it}k_{it}(z)$$

  s.c. $F_{it}(z) = k_{it}^{\alpha}h_{it}^{1-\alpha}(z) \geq T$

  After some calculation I obtain the nominal total production cost:

  $$CT_{it}(z) = P^C_{it}F_{it}(z) \left[ \left( \frac{1-\alpha}{\alpha} \right)^{\alpha} z_{it}^{\alpha}w_{it}^{1-\alpha} + \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} z_{it}^{\alpha}w_{it}^{1-\alpha} \right]$$

  The unit cost is then:

  $$Cm_{it}(z) = P^C_{it} \left[ \left( \frac{1-\alpha}{\alpha} \right)^{\alpha} z_{it}^{\alpha}w_{it}^{1-\alpha} + \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} z_{it}^{\alpha}w_{it}^{1-\alpha} \right]$$

  or

  $$Cm_{it}(z) = P^C_{it}z_{it}^{\alpha}w_{it}^{1-\alpha}(1-\alpha)^{\alpha-1}$$

  Labor and capital demand are

  $$k_{it}(z) = \frac{\alpha Cm_{it}(z)}{z_{it} P^C_{it}} F(k_{it}(z), h_{it}(z))$$  \hspace{1cm} (40)

  $$h_{it}(z) = \frac{1-\alpha Cm_{it}(z)}{w_{it} P^C_{it}} F(k_{it}(z), h_{it}(z))$$  \hspace{1cm} (41)
Second step: Optimal price and production

The objective is to maximize the discounted stream of dividends payments given the demand function for good \( z \) in each country and the law of motion of capital. The program is written as a Bellman equation:

\[
V(k_{1t}(z), p_{1t-1}(z), p^*_1(z)) = \max \left\{ \begin{array}{l}
  p_{1t}(z)x_{1t}(z) + c_1 p^*_1(z) x^*_1(z) + P^C_{2t} z_{1t}(z) - CT_{1t}(z) \\
  -(1 + R_{1t}) P^C_{2t} I_{1t}(z) - P^C_{2t} \frac{k_{1t+1}(z) - k_{1t}(z))^2}{2} \\
  + \int \chi(s_{t+1})V(k_{1t+1}(z), p_{1t}(z), p^*_1(z))ds_{t+1}
\end{array} \right\}
\]

subject to

\[
\begin{align*}
  k_{1t+1}(z) &= (1 - \delta) k_{1t}(z) + I_{1t}(z) \\
  x_{1t}(z) &= \left[ \frac{p_{1t}(z)}{P^C_{2t}} \right]^{-\eta} D^C_{1t} (\nu_{1t}(z)) \\
  x^*_1(z) &= \left[ \frac{p^*_1(z)}{P^C_{2t}} \right]^{-\eta} D^C_{2t} (\nu^*_1(z))
\end{align*}
\]

The first order conditions for \( x_{1t}(z) \) and \( x^*_1(z) \) give

\[
\begin{align*}
  p_{1t}(z) - \nu_{1t}(z) &= C m_{1t}(z) \\
  c_1 p^*_1(z) - \nu_{1t}(z) &= C m_{1t}(z)
\end{align*}
\]

Mark-up rate expressions

Given the mark-up rate definition I obtain

\[
p_{1t}(z) = (1 + \mu_{1t}(z)) C m_{1t}(z) = (1 + \mu_{1t}(z))(p_{1t}(z) - \nu_{1t}(z))
\]

which finally gives the mark-up expression

\[
\mu_{1t}(z) = \frac{\nu_{1t}(z)}{p_{1t}(z) - \nu_{1t}(z)}
\]

Similar expression can be obtained for the mark-up rate set for the foreign market \( j \) \((j \neq i)\). Optimal demand for capital and labor thus simplify:

\[
\begin{align*}
w_{1t} &= \frac{1}{1 + \mu_{1t}(z)} \frac{p_{1t}(z) x_{1t}(z) + x^*_1(z)}{P^C_{1t} h_{1t}(z)} \\
z_{1t} &= \frac{1}{1 + \mu_{1t}(z)} \frac{p_{1t}(z) x_{1t}(z) + x^*_1(z)}{k_{1t}(z)}
\end{align*}
\]
B Transition dynamics in the pricing-to-market model (M1)

In this section I present the consequences of a 1% increase in the domestic money growth factor in terms of impulse response functions.

Figure 9 presents the impulse response functions of aggregate real variables following a positive domestic monetary shock, that is consumption, employment, output and investment.

Figure 9: Real variables impulse response functions

Figure 10 displays the impulse response functions of marginal wealth and aggregate demand following a 1% increase in the domestic money growth factor. It also displays the response of quantities produced by individual firms, whether they are from country 1 or 2 and whether they price-to-market or not.

Finally, figure 11 presents the impulse response functions of the mark-up rates with the domestic monetary shock.
C Transition dynamics in model 2

This section presents the transition dynamics of variables following a 1% increase in the domestic money growth rate in model 2. Figure 12 presents the impulse response functions of aggregate real variables following a positive domestic monetary shock in the final model.

Figure 11 presents the impulse response functions for marginal wealth, aggregate demand and individual output, following the domestic monetary shock given PTM and limited participation.

Finally, figure 14 presents the impulse response functions for mark-up rates and sale prices.
Figure 11: Mark-up rates impulse response functions

Figure 12: Real variables impulse response functions
Figure 13: Wealth, demand and individual output

Figure 14: Mark-up rates and sale prices