Inside Money, Debt, and the Mechanics of Exchange

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Abstract

When agents are not trustworthy, the use of inside money can improve on private debts arrangements. Incentives to repay one's debt depend on the long term value of the ongoing relationship. Inside money is issued by a bank upon the request of agents, and the incentives to reimburse the bank depend on the value of future access to bank money. When bilateral interaction are weak, the mere threat of being excluded from the future access to money is sufficient to support exchange. Long term relationships between agents are replaced by a long term relationship with the bank.

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1 Introduction

This paper shows that in an environment with limited opportunities for enforcing contracts, *inside money* issued by a bank upon the request of private agents can be essential. The search theoretic approach to monetary economics (Kiyotaki and Wright, 1989; Williamson and Wright, 1994) specifies environments where the circulation of *fiat money* can improve on barter. Money is essential, *viz* allows to implement superior allocations. The venerable questioning of Menger (1892) on the acceptability of barren token has therefore received a formal treatment, based on the absence of double coincidence of wants or on doubts about the quality of goods. Attention has then been focused on determining the specific assumptions making money essential. Both lack of commitment and imperfect record-keeping appear to be necessary. Noting that *search*, OLG and *turnpike* models all share those features, Kocherlakota (1998*b*) argues that money acts as a record keeping device and a substitute for collective memory[1].

Yet two others aspects of these models, which have not been discussed so far, deserve further scrutiny. On the one hand, attention is confined to *outside* money. This imposes a *cash in advance* constraint, whereby money must be obtained through previous exchanges. Therefore, the holding of those token conveys information about prior behavior, which justifies the *social memory* interpretation (Townsend, 1989; Kocherlakota, 1998*b*). On the other hand, the use of loans is ruled out by the environment. At the same time, a simple observation of modern monetary economics reveals the three following features. First, most circulating money represent claims on the banking system. Second, money balances can be obtained through bank credit and money creation (credit card) Finally, loan arrangements account for a non negligeable share of transactions. This raises two questions. Does the inside / outside nature of the object called money matters for its essentiality? Is *cash in advance* a necessary condition for money to be essential?

This paper answers both question in the negative. I consider an environment with limited commitment and imperfect record keeping, and show that the enforcement of intertemporal trade can be delegated to a bank with a monopoly power to issue

[1]Recent contributions include Kocherlakota (2002), Temzelides and Yu (2001) and Araujo (2001) Wallace (2001) discusses the implications for macroeconomic monetary theory. For earlier related work see Ostroy and Starr (1974) on information, and Gale (1982) on the problem of trust (especially chap. 6).

money upon the request of agents. Therefore, my focus is on the way agents can obtain money. It is *inside* money because the claim on the bank is counterbalanced by the debt of the agent that obtained money (Gurley and Shaw, 1960). A few papers study the circulation of private money issued by 'banks' (Williamson, 1999; Cavalcanti and Wallace, 1999*a*, 1999*b*). Private money is inside money as the IOU cancels out with the debt of the issuer. But non bank agents are subject to the afore-mentioned *cash-in-advance* constraint, as the IOU must be obtained through previous exchange with a bank[2].

Loans, outside money and inside money are three distinctive mechanisms to implement exchange. One important dimension in which they differ is in the relationships they involve between trading partners. Consider an environement where there is need for intertemporal trade, and no commitment. Agents expect to trade with each other frequently[3]. First consider that private IOUs are used as a means of exchange:

Private debt

$$t=1 \qquad\qquad\qquad\qquad t=2$$

$$A \underset{\xleftarrow[\text{IOU}_B]{}}{\xrightarrow{\text{good}}} B \qquad\qquad A \underset{\xrightarrow[\text{IOU}_B]{}}{\xleftarrow{\text{good}}} B$$

A gives up his good at $t=1$ in exchange for a promise by B to deliver another good in $t=2$, represented by B's *IOU* The intertemporal nature of the agreement raises the question of the incentive for B to pay off his debt[4]. With perfect information in the sense of perfect record-keeping of past transactions, the collective threat of autarky if B deviates can be sufficient to enforce exchange (Kocherlakota, 1998*a*). However, this result hinges on perfect record keeping *and* on the ability of agents to coordinate on the equilibrium with the largest threat[5]. This situation is akin to a

[2] In Bullard and Smith (2001) outside money is issued by the government, and inside money by private agents. This identification with the nature of the issuer is not implied by the definition of Gurley and Shaw (1960). For inside money issued by a government bank, see for example Freeman (1996) and Dubey and Geanakoplos (2000).

[3] The economy to be considered is finite, which insures that agents have a positive probability of interacting again in the future. This is a departure from *search* framework.

[4] Goodhart (1989) makes the point that the lack of personal information on the credit standing of potential buyers leads society to use a means of paiment rather than to rely on loan arrangements.

[5] Kocherlakota's (1998*b*) results rely on the game of exchange being repeated at the level of society. The multiplicity of Nash equilibria follows from a standard folk theorem argument.

community. In a decentralized setting, agents only know their own trading history[6], and punishment is bound to be bilateral: A can refuse any future exchange with B. The amount of exchange that can be supported with private debt is therefore limited by the value of maintaining a trade relationship. The more agents discount future consumption, or the larger the economy, the more inefficent is debt in supporting exchange (section 3). The discussion of the cost of an enduring relationship is sidestepped to section 8.

Outside money

$$t = 1$$

$$A \quad \underset{\text{m}}{\overset{\text{good}}{\rightleftarrows}} \quad B$$

How does outside money mitigate the no commitment problem? When A exchange with B, his sole question is to evaluate the future acceptability of money. If money has value in equilibrium, exchange takes place. Outside money thus differs significantly from bilateral loans in that no intertemporal relation between individual is needed. Bernhardt (1989) and Townsend (1989) both study environment where outside money and credit coexist. As money balances comes from previous exchanges, the use of money acts as if it signalled that B has produced in the past for society, and is now entitled to consumption. In this case, exchange is limited by the money balances of the potential buyer (Bernhardt, 1989). Imposing a *cash in advance* constraint does more than stating that money is used in all transaction. It states that money must be obtained through prior exchange.

Inside money

$$\begin{array}{cccc} \text{bank} & & & \text{bank} \\ \text{m} \downarrow\uparrow \text{IOU}_A & & & \text{IOU}_A \downarrow\uparrow \text{m} \\ A & A \underset{\text{good}}{\overset{\text{m}}{\rightleftarrows}} B & A \underset{\text{m}}{\overset{\text{good}}{\rightleftarrows}} B & A \end{array}$$

Now consider how inside money might work. The above diagram shows the relevant sequence of events. Suppose that agent A goes to the bank to obtain money - unfalsifiable paper bearing the bank's mark - against the promise to give money back in the future. This corresponds to IOU_A being given to the bank in exchange

[6] Ostroy and Starr (1974) have insisted on this point.

for m. Now A can exchange money for goods with B. Why should B stand ready to give up his good for a useless paper m? Because he expects to trade with it. This occurs if B meets an agent (say A again) willing to accept money because he needs it to pay off the bank. Another question then arises: How can the bank get its piece of paper back? One could assume that the bank has a coercitive power to enforce contracts, but this would amount to assuming what must be proven. With no commitment, A pays off his debt if and only if he has an incentive to do so. Suppose the following: if A renege on his debt towards the bank, he is refused the access to m in the future. Money works as a kind of collateral with value the rent associated with future access to money. By excluding A from the access to money, the bank punishes him on behalf on society. The enforcement of debt is thus delegated to the bank. The relationship between A and B is replaced by a long term relationship with the bank. Money acts as a self-binding mechanism whereby A gives as a collateral the value associated with the access to m in each future period. This result is formally proved in section 5.

The remainder of the paper is organised as follows. Section 2 presents the environnement and the first best equilibrium. Section 3 characterizes the equilibrium with private debt, and section 4 the equilibrium with money. The main findings are presented and discussed in section 5. Section 6 shows how the analysis is modified when access to money is costly. Section 7 analyses case 2. In section 8, I allow agents to form tight relationships and illustrate why money could still be essential. A brief conclusion is drawn in 9.

2 The environment

Time is discrete, and continues forever; $t = 1, 2, ..., \infty$. The economy is populated by $2N$ agents, N entrepreneurs and N investors. Each agent has the same utility function, given by

$$\sum_{t=1}^{\infty} \delta^t u(c_t)$$

Instantaneous utility is taken to be linear, $u(c) = c$. Think of each period as being subdivided into two subperiods, denoted t^- and t^+. In each period,, an investor receives in t^- an endowment $e_t = 1$ of a divisible input good. Investors have access to a storage technology transforming 1 unit of the input good into R unit of consumption good in t^+. All goods fully depreciate between t^+ and $(t+1)^-$. Entrepreneurs have no endowment, but have access to a production technology $f(x)$ that produces the end of period consumption good. The production technology satisfies usual regularity conditions[7], plus $f'(1) \geq R$ (implying $f'(x) > R$ for all $x < 1$). Applications will use the functional form

$$f(x) = Ax^a \qquad \text{with } 0 < \alpha < 1 \text{ and } A \geq \frac{R}{\alpha}$$

Entrepreneurs cannot consume without exchanging. Entrepreneurs and investors interact as follows. There is an archipelago composed of one central island and N outlying islands, indexed $i \in \{1, .., N\}$. In a given period, every investor $k \in \{1, .., N\}$ is located on a distinct outlying island. For the ease of exposition, I use a slightly more general notation than necessary. Investor's location at t can be described by $\sigma_t(k) = i$, $k = 1, .., N$. σ_t is a bijection from $\{1, .., N\}$ into $\{1, .., N\}$. These locations are not publicly observed. All public information is that investor's locations follows a stochastic process $\{\tilde{\sigma}_t\}_0^\infty$. The realisations of $\tilde{\sigma}_t$ are not publicly observed. In t^+, each producer chooses an island[8], using any private information he has on investor's location (from his own trading history).He travels to this island, learns the identity of the investor, gets some input x, and comes back to the central island where they produce (say, because some essential tools are there). Each producer then knows the identity of the investor on the island he has visited. In t^+ he can then make a end of period trip to the same island to pay off his debt. Consumption takes place, and investors are eventually shifted to new locations.

The following diagram sums up the sequence of event that take place within one period:

t					$t+1$
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
endowments	1^{st}trip x	production $f(x)$	2^{nd} trip	consumption d, c	$\tilde{\sigma}_{t+1}$

[7] Except when f is taken to be linear, f will satisfy $f(0) = 0$, $f'(x) > 0$ and $f''(x) < 0$, and $\lim_{x\to 0} f'(x) = +\infty$.

[8] Compatibility of producers'strategy requires that each one is assigned a distinct island. This requirement is satisfied in the equilibria to be considered.

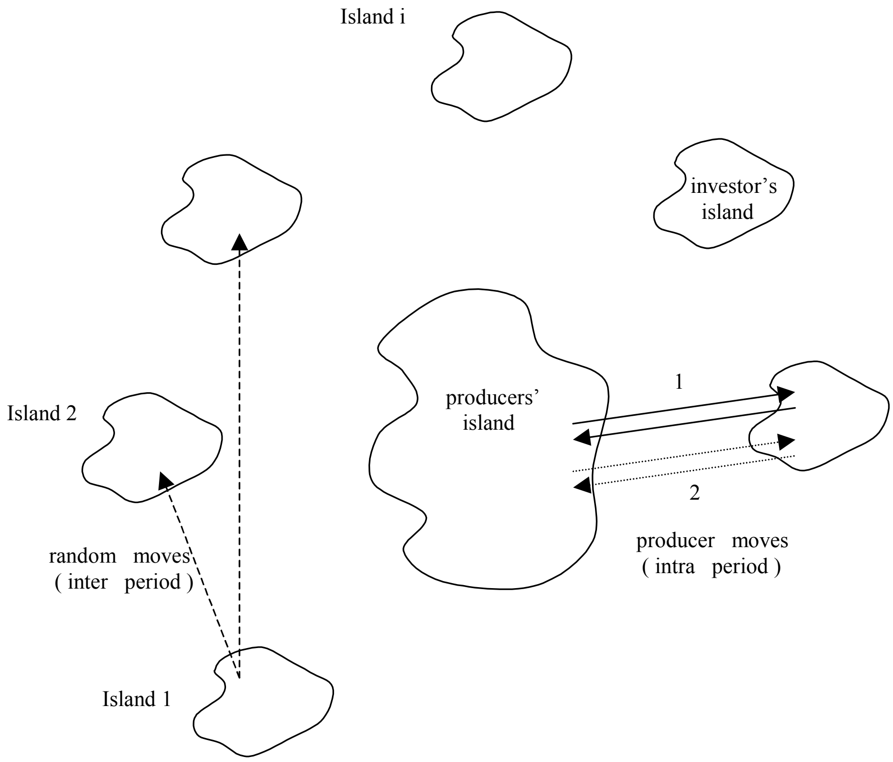

Figure 1: The archipelago

I will restrict attention to two cases:

Case 1. $\tilde{\sigma}_t = \tilde{\upsilon}\ \forall t$, *denoting by* $\tilde{\upsilon}$ *the uniform distribution on investor's possible location*[9]*..*

Case 2. $\{\tilde{\sigma}_t\}_0^\infty$ *follows a Markov process* $\tilde{\sigma}_{t+1} = \theta\sigma_t + (1-\theta)\,\tilde{\upsilon}$*, with* $0 \leq \theta \leq 1$*. With some probability* θ*, investors stay on their past location. With probability* $1-\theta$ *they are shifted at random on any island*[10]

Apart from section 7, we will confine attention to the simple case, $\tilde{\sigma}_t = \tilde{\upsilon}\ \forall t$. Formally, this is akin to a pairwise random matching with pairs lasting for one period, and exchange, production and consumption occuring during this period. The probability for a given entrepreneur j to be matched with a given investor j is then $\frac{1}{N}$, which is a natural measure of the strengh of interactions.

[9] This is easy to define, as the set of bijection from $\{1,..,N\}$ into itself is finite.

[10] Note that the probability for a given investor to stay on his location should take into account the probability to draw this island conditional on drawing, $\Pr\left[\sigma_t(k) = \sigma_{t-1}(k)\right] = \theta + (1-\theta)\,\frac{1}{N}$.

As for general notation, let c denote the investor's consumption and d the entrepreneur's consumption. In all that follows, it is assumed that the entrepreneur has all the bargaining power. As a benchmark, first suppose there is no commitment problem, so that any agreed promise made in t^- is fulfilled in t^+. Any arrangement must then satisfy two feasability conditions - one per subperiod - and the investor's participation constraint:

$$\begin{aligned} x &\leq 1 \\ c + d &\leq f(x) \\ c &\geq Rx \end{aligned} \tag{2.1}$$

When commitment is not a problem, given that the marginal rate of transformation is superior to R for $x \leq 1$, one trivially have

Proposition 2.1. *It is optimal to have all input used in production, $x^* = 1$. As entrepreneurs get all the surplus from exchange, $c^* = R$ and $d^* = f(1) - R$*

3 Exchange under private debt

I now introduce the commitment problem. The optimal arrangement requires that, in each period, investors give up x^* in exchange for a promise (be it verbal or represented by an IOU) to get Rx^* in t^+. This raises the following question: Will the entrepreneur, after production, have an interest to fullfil his promise to the investor he has just exchanged with. If he reneges on his promise, he can consume the entire output $f(x)$, instead of $f(x) - Rx$. But this comes at the cost of loosing any future exchange with k. Credit is time consistent insofar as the rent associated to future bilateral exchange with k outweighs the immediate gain from reneging. Otherwise, investors would refuse to exchange in the first place.

What is the value of a trading relationship? Suppose that entrepreneur's consumption is d in each encounter. From the entrepreneur's point of view, the rent associated with this relationship can be formally writen as:

$$V = \mathrm{E}\left[\sum_{n=1}^{\infty} \delta^{T_n} u(d)\right] = H(\delta, N) \cdot d \tag{3.1}$$

where $\{T_n\}_{n=1}^{\infty}$ is the set of encounters with k, depending on the stochastic process[11]

[11] Define $T_0 = 0$ and $T_{n+1} = \inf_{t>T_n} \left\{t|\sigma_t = s_t^j\right\}$, with s_t^j the island chosen by entrepreneur j at date t.

$\{\tilde{\sigma}_t\}_{t=1}^{\infty}$. More precisely, $H = \mathrm{E}\left[\Sigma\varepsilon(t)\,\delta^t\right]$ with $\varepsilon(t) = 1$ if j chooses at t the island where investor k is located. As history conveys no information on investors' location, the $\varepsilon(t)$ are independent and $\mathrm{E}[\varepsilon(t)] = \frac{1}{N}$. It follows that $H(\delta, N) = \mathrm{E}\left[\Sigma\varepsilon(t)\,\delta^t\right] = \frac{1}{N}\frac{\delta}{1-\delta}$. Another way to compute $H(\delta, N)$ consists of using the fact that there are N potential partners, with identical meeting probability that sum to 1. Hence $NH = \Sigma_{t=1}^{\infty}\delta^t = \frac{\delta}{1-\delta}$, which yields H. Unsurprisingly, the higher the discount factor δ or the smaller the population, the higher the horizon H.

If contracts could be costlessly enforced by a *deus ex machina*, one would have $x^* = 1$. This optimum is implementable if and only if debt satisfies the incentive constraint

$$u(f(x^*)) - u(d^*) < H(\delta, N)\,u(d^*) \tag{3.2}$$

or

$$f(1) < (H(\delta, N) + 1)\,d^* \tag{3.3}$$

When condition (3.2) is violated, k has an incentive to break the agreement, i.e. he will be better off if he keeps the entire output at the end of period t.

Proposition 3.1. *The first best allocation is not attainable when 1/ the economy is too large or 2/ the degree of time preference is too high.*

Proof. Follows directly from the monotonicity of $H(\delta, N)$ and some limit properties. $H(\delta, N)$ increases with δ and decreases with N. Moreover, $H(0, N) = 0$, $\lim_{\delta\to 1} H(\delta, N) = +\infty$. Finally $\lim_{N\to 1} H(\delta, N) = 0$. □

What happens when condition (3.2) is not satisfied? Intuition suggest the following:

Conjecture 3.2. *Exchange is characterized by*

$$x = \sup_{0\le x\le 1}\{x | u(f(x)) < (H(\delta, N) + 1)\,u(d(x))\}$$

$d(x) = f(x) - Rx$ *being the entrepreneur's consumption when a quantity x is exchanged.*

For exchange to be time consistent, the entrepreneur's offer must satisfy the following incentive condition

$$Rx < H(\delta, N)\, d^{+} \tag{3.4}$$

where d^{+} is j's consumption in future encounters with k. Condition (3.4) says that the rent associated with the trading relationship, $H(\delta, N)\, d^{+}$, outweighs the immediate gain from reneging, Rx. In equilibrium, $d^{+} = d$, so that eq. (3.4) can be rewritten as

$$Rx < H(\delta, N)\,(f(x) - Rx) \tag{3.5}$$

As j's consumption increases with the amount of input exchanged x, and given that the investor just breaks even, the conjecture is correct. The entrepreneur will then offer the arrangement (x, d, c) that maximizes consumption subject to

$$\begin{aligned} x &\leq 1 \\ c + d &\leq f(x) \\ c &\geq Rx && (3.6) \\ f(x) - d &\leq H(\delta, N)\,(f(x) - Rx) && (3.7) \end{aligned}$$

Appart from the feasability constraints and the investor's participation constraint, exchange must now satisfy the incentive constraint (3.7). Condition (2.1) reduces to

$$Rx \leq \frac{H(\delta, N)}{1 + H(\delta, N)} f(x) \tag{3.8}$$

The following proposition fully characterizes the equilibrium attainable under privet debt arrangement.

Proposition 3.3. *For a standard production function (satisfying Inada's conditions), there exists $\varepsilon > 0$ such that*

1. *Private loans implements the first best allocation iff $H(\delta, N) > \varepsilon$;*
2. *If $H(\delta, N) < \varepsilon$, the equilibrium under private credit is characterized by*

$$\begin{aligned} \frac{f(x^c)}{x^c} &= \frac{1 + H(\delta, N)}{H(\delta, N)} R && (3.9) \\ d^c &= f(x^c) - Rx^c = \frac{R}{H(\delta, N)} x^c \\ c^c &= Rx^c \end{aligned}$$

In this case, x^c and d^c are increasing function of $H(\delta, N)$.

Proof. (x^*, d^*, c^*) is attainable iff $f(x^*) < (H(\delta, N) + 1) d^*$, which yields $H(\delta, N) > \frac{R}{f(1)-R} \equiv \varepsilon$. The second part follows from the fact that $\arg\max \{f(x) - Rx\} > 1$. $\square$

Denote by $x^c(\delta, N)$ and $d^c(\delta, N)$ the solution to (3.9). The inefficiency stemming from the no commitmemt problem can be measured by $x^* - x^c(\delta, N)$. When $x^c(\delta, N) < x^*$, the amount of trade and the consumption of entrepreneur rises as δ rises or as N decreases. Hence, stronger interactions - due either to a fall in the rate of time preference ($\delta \nearrow$) or a fall in population size ($N \searrow$) - allows more exchange. For the Cobb-Douglas specification, one has

Corollary 3.4. *For $f(x) = Ax^\alpha$, the threshold is*

$$\varepsilon = \frac{R}{A - R}$$

and the constrained equilibrium is characterized by

$$x^c = \left(\frac{H(\delta, N)}{1 + H(\delta, N)} \frac{A}{R} \right)^{\frac{1}{1-\alpha}}$$

x^c increases with $H(\delta, N)$. The instantaneous consumption of an entrepreneur is given by

$$d^c = \left(\frac{A}{1 + H(\delta, N)} \left(\frac{H(\delta, N)}{R} \right)^\alpha \right)^{\frac{1}{1-\alpha}}$$

One can easily check that d^c increases with $H(\delta, N)$ for $H(\delta, N) < \varepsilon$.

Proof. Computations are straightforward, and hence omitted. $\square$

4 Exchange with bank money

Supppose now that there is one bank, located on the central island. The distinctive feature of this agent is its monopoly in the issuing of m. Assume that this bank stands ready to issue money (IOU) upon the request of entrepreneurs, provided that it anticipate that entrepreneurs will pay off their debt, in a sense that will be made precise latter.

$$\begin{array}{c} \text{bank} \\ \text{m} \downarrow\uparrow \text{IOU}_j \\ \text{entr.}j \end{array} \qquad \text{entr.}j \underset{\text{x}}{\overset{\text{m}}{\rightleftarrows}} \text{inv.}k \qquad \text{entr.}j \underset{\text{m}}{\overset{\text{c}}{\rightleftarrows}} \text{inv.}k \qquad \begin{array}{c} \text{bank} \\ \text{IOU}_j \downarrow\uparrow \text{m} \\ \text{entr } j \end{array}$$

From now on, suppose that the bank's bargaining power is zero. Section 6 introduces an interest on credit money. At the beginning of period t, and prior to his trip, entrepreneur j obtains an indivisible piece of paper m from the bank. Now upon meeting an investor, he can offer the following deal. He takes x and gives up m with the promise to offer Rx in subperiod t^+ to get m back. After production, j goes back to island i if and only if he has an incentive to.pay off his debt to the bank. If he does not reimburse, he will be excluded from the access to bank money in the future. To determine whether j has an incentive to fulfil the agreement made with the bank, one must determine his fallback position. Whereas the identity of j appears on his own IOU, only the bank's mark appears on m. It thus makes sense to assume that investors no longer punish a producer. The threat of punishment is fully delegated to the banking system. Denote by d^b the amount of consumption with this bank mechanism. If j deviates, his instantaneous payoff is $\Delta u_0 = u(f) - u\left(d^b\right)$, and he will be limited in future transactions to the use of debt, yielding d^c in each period. The acceptability of m in t^+ is the incentive constraint for entrepreneurs. A necessary condition for inside money to circulate is thus

$$\Delta u_0 + \sum_{t=1}^{\infty} \delta^t u\left(d^c\right) < \sum_{t=1}^{\infty} \delta^t u\left(d^b\right) \tag{4.1}$$

$$\underbrace{f(x) - d}_{\text{immediate gain}} \quad + \quad \underbrace{\frac{\delta}{1-\delta} d^c}_{\text{fallback position}} \quad < \quad \underbrace{\frac{\delta}{1-\delta} d}_{\text{monetary rent}} \tag{4.2}$$

Note that private credit restricts the exchange attainable with inside money. Eq. (4.2) rewrites as

$$f(x) - d < \frac{\delta}{1-\delta}\left(f(x) - Rx - d^c\right) \tag{4.3}$$

or, alternatively

$$Rx < \delta \left(f(x) - d^c\right) \tag{4.4}$$

Noting that $\frac{\delta}{1-\delta} = H(1, \delta)$, one can express (4.3) in a way similar to eq. (3.8):

$$Rx \leq \frac{H(1,N)}{1 + H(1,N)} \left(f(x) - d^c\right) \tag{4.5}$$

5 Main results

The first best allocation can be implemented with private debt if and only if

$$u(f) - u(d^*) < H(\delta, N)\, u(d^*) \tag{5.1}$$

and with bank money if and only if

$$u(f) - u(d^*) + \sum_{t=1}^{\infty} \delta^t u(d^c) < \sum_{t=1}^{\infty} \delta^t u(d^*) \tag{5.2}$$

Proposition 5.1. *One can construct cases where first best is attainable under inside money, but not with private debt.*

Proof. Take a linear technology $f(x) = Ax$, $A > R$. Condition (5.1) then gives $Rx < H(\delta, N)(A - R)x$, When $R > H(\delta, N)(A - R)$, debt supports no trade, $x^c = 0$. This occurs for N high enough. Then $d^c = 0$ and inside money implements first best iff $R < \delta A$. □

More generally, for inside money to enforce intertemporal trade, private debts need to be sufficiently inefficient. Recall that with debt the time consistency of exchange requires:

$$Rx < \frac{H(\delta, N)}{1 + H(\delta, N)} f(x) \tag{5.3}$$

whereas the counterpart with inside money is the incentive to get money back:

$$Rx < \frac{H(1,N)}{1 + H(1,N)} \left(f(x) - d^c\right) \tag{5.4}$$

The following proposition contains the main result of the paper.

Proposition 5.2. *Define $\hat{x}$ by $\delta f'(\hat{x}) = R$. There exist an implementable allocation under bank money if and only if*

$$f(\hat{x}) - \hat{x} f'(\hat{x}) > d^c$$

In this case, bank money dominates private loans.

Proof. Let $F(x) = Rx - \frac{H(1,N)}{1+H(1,N)}(f(x) - d^c)$. Condition (5.4) writes $F(x) < 0$. Using, $\frac{H(1,N)}{1+H(1,N)} = \delta$. $F'(x) = R - \delta f'(x)$. $F(.)$ is convex, $F'(0) = -\infty$, and $\lim_{+\infty} F' = R > 0$. F reaches a minimum on the interior of $\Re$, say at $\hat{x}$, given by $\delta f'(\hat{x}) = R$. Bank money circulates iff $\min F(x) = F(\hat{x}) < 0$. But $F(\hat{x}) = R\hat{x} - \delta(f(\hat{x}) - d^c) = \delta(\hat{x}f'(\hat{x}) - f(\hat{x}) + d^c)$. This yields the condition. As for second part, it suffices to show that $\hat{x} > x^c$ when the above conditon is satisfied. We can rewrites $F(x) = (1-\delta) Rx - \delta(f(x) - Rx - d^c)$. The function $G(x) \equiv f(x) - Rx - d^c$ is increasing on $[0, x^c]$ and $G(x^c) = 0$. Therefore $x < x^c$ implies $F(x) > 0$, and thus if $F(\hat{x}) < 0$ then $\hat{x} > x^c$. □

Corollary 5.3. *Take $f(x) = Ax^\alpha$. Inside money circulates iff*

$$(1-\alpha)^{1-\alpha} (\delta\alpha)^\alpha > \frac{H(\delta,N)^\alpha}{1 + H(\delta,N)}$$

Proof. $\delta f'(\hat{x}) = R$ yields $\hat{x} = \left[\delta\alpha\frac{A}{R}\right]^{\frac{1}{1-\alpha}}$ Money circulates iff $(1-\alpha) A\hat{x}^\alpha > d^c$. Using the expression for d^c and rearranging, one gets the condition stated. □

The logic behind this result is simple, and is illustrated in figure 2. The bank punishes a deviant producer by excluding him from money (m) in the future. The entrepreneur then looses at each date the instananeous gain associated with the access to m, i.e. the increment it allows over exchange with debt. As the size of the population grows, the value of a given bilateral relationship shrinks, which lowers d^c and raises the value of the access to the bank piece of paper. Inside money issued by the bank is thus viable as an enforcement mechanism insofar as private debt does bad enough. Inside money thus circulate if and only if

$$f(\hat{x}) - \hat{x}\frac{R}{\delta} > d^c \tag{5.5}$$

Combining previous results yields the following proposition, which illustrated by figure 3

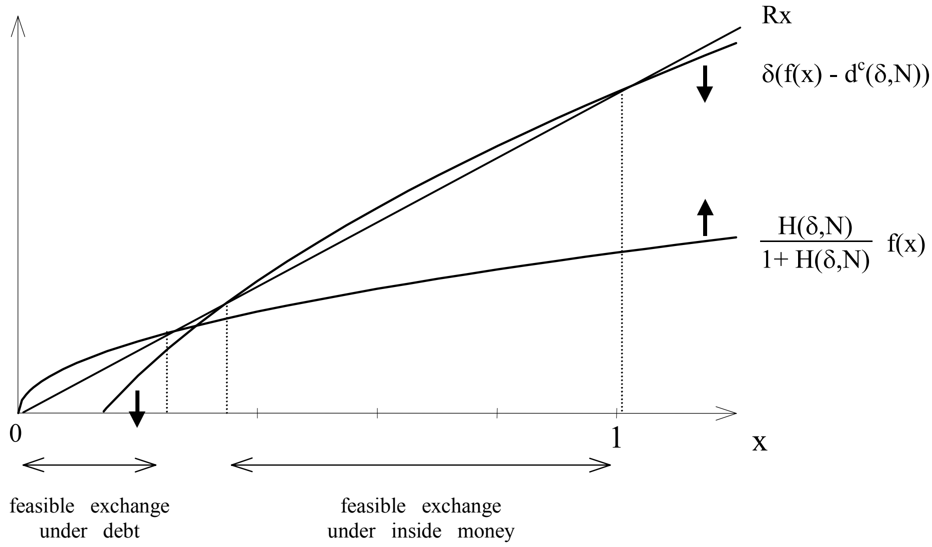

Figure 2: Ranges of x satisfying incentive constraints (5.3) and (5.4). Thick arrows indicate the effect of a fall in the size N Parameters values are $R = 1.1$, $\alpha = 0.5$, $A = 2.2$, $\delta = 0.82$, $N = 20$.

Proposition 5.4. *Let δ be such that $H(\delta, 1) > \varepsilon$. Then there exist $N_c < N^* < N_b$ such that:*

- *For $N < N_c$, private debt are used and implement the first best arrangement;*
- *For $N_c < N < N^*$, private debts are used but cannot support x^**
- *For $N^* < N < N_b$, inside money circulates and x^b is strictly less than x^*;*
- *For $N_b < N$, inside money circulates and supports the first best arrangement.*

Figure 3 is drawn with the following example:

Example 1. $f(x) = A\sqrt{x}$.*The equilibrium with private debt is*

$$x^c = \max\left\{\left(\frac{1}{1+N\left(\frac{1}{\delta}-1\right)}\frac{A}{R}\right)^2, 1\right\}$$

Under inside money, eq. (4.4) writes $Rx = \delta\left(Ax^{\frac{1}{2}} - d^c\right)$ x^b is the largest root, when it exists. Hence

$$x^b = \left[\frac{\delta A + \sqrt{\delta^2 A^2 - 4\delta R d^c}}{2R}\right]^2$$

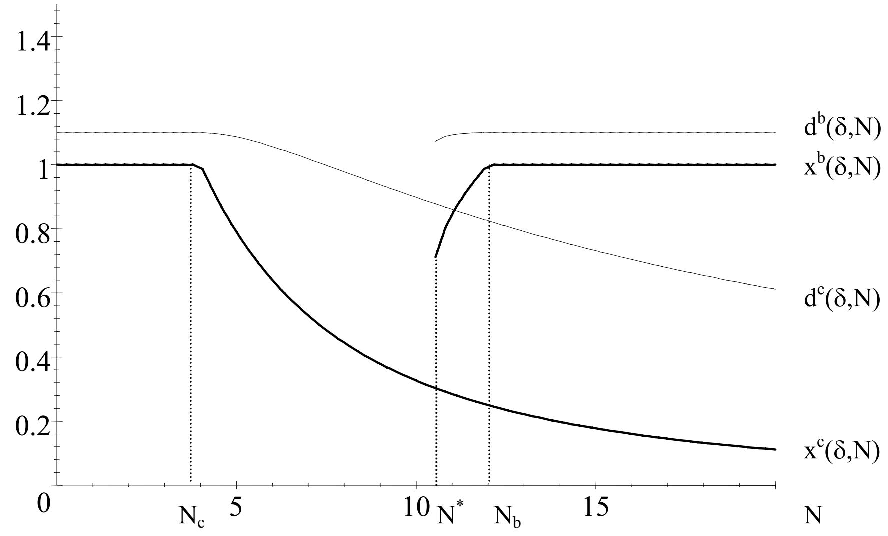

Figure 3: Exchange mechanism and amount of trade depending on the population size (proposition 5.4). $R = 1.1$, $\alpha = 0.5$, $A = 2.2$, $\delta = 0.82$

with the condition $\delta^2 A^2 - 4\delta R d^c > 0$, *yielding* $\delta \frac{A}{R}\frac{1}{4} > d^c$.

Inside money works as follows. The use of m by the entrepreneur increases his ability to commit to intertemporal exchange. By giving up money, the entrepreneur puts some kind of collateral in the intertemporal agreement. This 'collateral' is the rent associated with the access to money creation. One might wonder what are the crucial assumptions that allows inside money to improve on individual IOUs. The bank is easy to locate, and has monopoly in the issuing of m. There is no higher probability to meet the bank, and the monetary institution has no special power to enforce contracts. An entrepreneur can hide from the bank, and exchange in the future through private debt. As section 7 will make clear, the easiness to be located is not the crucial feature. The important assumption is that upon reneging on his obligation towards the banking system, the entrepreneur is refused access to money (liquidity) in the future. In a banking system with different banks, what is needed is some sharing on information about people that renege on their debt[12]. Gale (1982) offers an interesting analysis of how the issuing of money can improve exchange in the absence of trust. Nevertheless, the value of money is backed by the power of the

[12]In the french system, the empirical counterpart is the 'interdit bancaire' procedure, whereby an agent can no longer have a credit card nor use a checking account *in any bank*.

state to levy taxes[13]. In my setting, the monetary authority is not endowed with any coercitive power, and redemption of the debt towards the bank faces the same limitation in commitment as private debt.

Proposition 5.4 makes another point that is worth noting. When inside money circulate without implementing the first best allocation, an increase in the efficiency of private contracts can be detrimental to welfare. Innovation in potential means of exchange exert pressure on monetary exchange.

6 Interest as the cost of the access to liquidity

So far, it has been assumed that the access to bank money was costless. This section shows how the results change when an interest on money is introduced. Suppose that when reimbursing the bank, entrepreneurs have to pay a quantity ρ of good. I take ρ to be exogenous, stemming either from the cost of the bank's technology, or from its bargaining power. The above analysis easily extends in this case with a few changes. The feasability constraint at t^+ becomes

$$f(x) = d + Rx + \rho \tag{6.1}$$

Using (6.1) to cancel d from (4.2) yields the following counterpart to (4.5), characterising implementable trade

$$Rx + \rho < \frac{H(1,N)}{1 + H(1,N)} \left(f(x) - d^c \right) \tag{6.2}$$

A brief look at the proof of proposition 5.2 shows that for inside money to be viable, one must have

$$f(\hat{x}) - \hat{x} f'(\hat{x}) > d^c + \frac{\rho}{\delta} \tag{6.3}$$

If condition (6.3) is satisfied, inside money circulates, as it implies that

$$d_b(\rho) \equiv f(\hat{x}) - R\hat{x} - \rho > d^c$$

[13] In Holmstrom and Tirole (1998), the ability of government to commit future income allows it to create liquidity when the private sector is unable to do so.

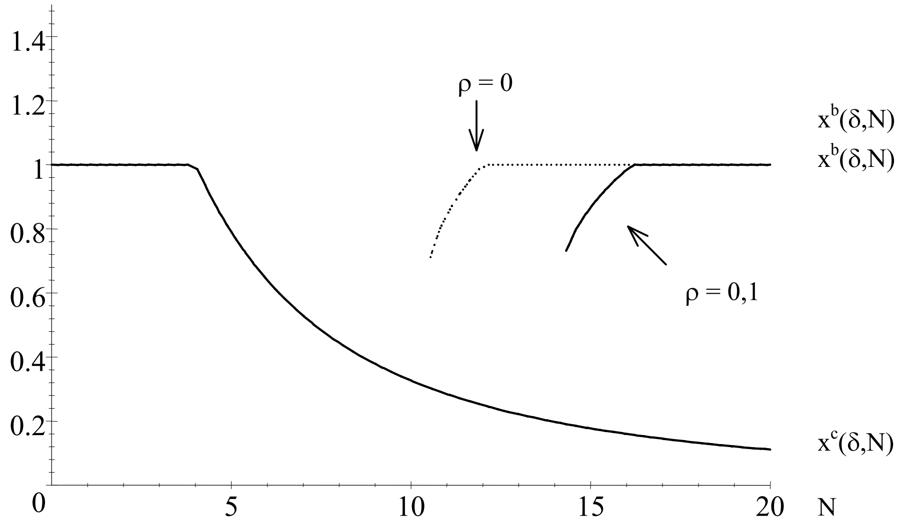

Figure 4: Exchange when ρ varies

Any event that raises the cost of issuing inside money can lead to an increment in ρ. One can think for instance of a tighter monetary policy or an adverse shock affecting the banking system. To illustrate possible implications, take $N = 15$ in figure 4 and start with $\rho = 0$. Now consider what happens when ρ rises. In a first time, nothing changes, until ρ reaches a level where exchange starts to diminish. Money still circulates, but $x < x^*$. Exchange diminishes continuously until a threshold for ρ when demonetisation occurs, and exchange drops suddenly.

7 Analysis of Case 2.

What are the features that prevent private debt from implementing the first debt allocation? One could argue that the inability to locate investors adds to the inability to commit. In fact, when entrepreneurs cannot commit to repay their debt, ability to trace investors (creditors) further hampers trade. The intuition is simple and runs as follows. Any deviant entrepreneur, in the sense that he renege on his promise towards a given investor k, will try to avoid k in the future. The information on k's location will then allow him to avoid k with a greater probability.

To formally prove this, I consider the case where locations exhibit some persistence. The memory of the identity of investors in the islands previously visited conveys an information on k's future position. The difficulty stems from the fact that this information will be used to meet / avoid this investor in future periods. Take an entrepreneur, say j, that trade with investor k at $t = 0$ on island i, and which is considering to renege in 0^+. The cost of reneging depends on future probabilities to meet k when optimally trying to avoid him. At any future date, the entrepreneur will use all available information (k was here / k was not there), and could even adopt a learning strategy. As a matter of fact, the optimal strategy proves to be intuitive and simple:

Lemma 7.1. *The optimum strategy to avoid investor k is the following. If j meets k at date t_1 on island i, then he chooses any island $i' \neq i$ in $t_1 + 1$, and stays there until he meets k again.*

Proof. See the appendix. □

Given this result, the maximum threat that an investor can make on k is easy to calculate. If k is on i_1 at t, he will stay there with probability $\theta + (1-\theta)\frac{1}{N}$. By choosing any other island $i_2 \neq i_1$, the probability that k is not there at $t+1$ is therefore $\theta + (1-\theta)\left(1 - \frac{1}{N}\right)$. Conversely, if j knows that k is not on a given island i_2 at t, he can infer that k will not be there at $t+1$ with probability $\theta + (1-\theta)\left(1 - \frac{1}{N}\right)$. Leaving i_2 would raise the probability of meeting k, as it is possible to choose the island were k was at $t+1$. In fact, the relevant information appears to be "k was not on island i at t". Following the best strategy, j can thus avoid investor k with probability $p \equiv \theta + (1-\theta)\left(1 - \frac{1}{N}\right)$ in each period. The future value abandonned by the entrepreneur is thus

$$V_\theta = [1-p]\,\frac{\delta}{1-\delta}d = \frac{1-\theta}{N}\frac{\delta}{1-\delta}d \tag{7.1}$$

One can check that for $\theta = 0$, V_0 reduces to $V = \frac{1}{N}\frac{\delta}{1-\delta}$. Taking the limit as θ tends to 1, V_θ tends to zero. Using the notation $H^\theta(\delta, N) = \frac{1-\theta}{N}\frac{\delta}{1-\delta}$, exchange is attainable if and only if $Rx \leq H^\theta(\delta, N)\,d$ which gives

$$Rx \leq \frac{H^\theta(\delta, N)}{1 + H^\theta(\delta, N)} f(x) \tag{7.2}$$

Ability to trace gives an advantage to the agents that can strategically use it. Nevertheless this is an *ex post* advantage that deteriorates *ex ante* the incentive constraint. Both the amount of exchange and welfare diminish as investors become easier to locate. This result naturally extends to more general processes (proof is omitted).

Corollary 7.2. *Any process* $\{\tilde{\sigma}_t\}_{t=1}^{\infty}$ *different from* $\tilde{\sigma}_t = \upsilon$ *leads to a shorter horizon than* $H(\delta, N)$.

8 Extension. Long term relationships.

When agents interact frequently, exchange can be supported by bilateral debt arrangements. So far, the strengh of these interactions have been exogenously given by the process $\{\tilde{\sigma}\}$. This suggests another way to support exchange. Suppose entrepreneurs have the ability to engage in long term relationship with a given investor. However, enforcement through close relationships can entail some cost, say because of renegociation or loss of competition. Most of these cost are opportunity costs. Money can still be essential, in this setting, for the use of money frees agents from developing close relationships, thus economises on the associated costs.

A complete analysis of how money and agents' relationships interact with each other is beyond the scope of this paper[14], and.I will only illustrate these intuitions in a very simple case. Take $f(x) = Ax$. Suppose that, upon meeting investor k, entrepreneur j can enter into a relationship with strengh $\chi \in (0,1)$: with probability χ the pair (j,k) will stay together next period[15], and is broken with probability $1-\chi$. If a pair is broken, j and k become strangers. Suppose that at each period, the entrepreneur receives a preference shock: for the current period j is only able to produce with the inputs of a proportion $1-\varepsilon$ of investors. Assume that j knows at the beginning of t^- the investors whom inputs he does not value. Then, with probability ε he does not value the endowment of the investor he was with. For exchange to take place, the following condition must be satisfied:

$$Rx \leq \frac{\delta\chi}{1-\delta\chi}(1-\varepsilon)(Ax - Rx) \tag{8.1}$$

Condition (8.1) states that the debt burden be less than the value attached to the ongoing relationship. Clearly, entrepreneur's choice of χ is given by

[14] Simmel (1977) have developed a philosophical analysis of how the use of money enhances freedom and anonimity.

[15] In a richer environment, χ could be thought as becoming relatives, or settling in a village, i.e. decisions that influence the future probability of interaction.

$$\chi^{*}=\frac{1}{\delta\left(1-\varepsilon\right)\frac{A-R}{R}+1} \tag{8.2}$$

What is the cost of this enduring relationship? It stems from the inability to transact with another investor when k's good is useless. Hence it can be expressed as $\chi^{*}\varepsilon$. With money, agents no longer need to enter into close relationship, and are free to choose any trading partner. As ε goes to 1, the opportunity cost rises, for tighter relationships must be formed to support exchange (χ^{*} raises). Keeping the analysis informal, one can conjecture that for ε high enough, money is a better mechanism for implementing exchange than forming close relationships.

9 Conclusion

This paper is connected to two literatures. The linkage with the literature on the rationale for money has been extensively discussed. This paper have shown that inside money created upon the request of agents can be essential, in an environment featuring the usual conditions for outside money to be essential. With inside money, the threat of exclusion from the access to money that the relationship with the bank offers can be sufficient to enforce intertemporal exchange. One consequence is that *cash in advance* contraint in the usual sense (money being obtained through trade at $t-1$) may not capture every equilibrium where money circulates. Although rudimentary, the model presented here provides an illustration on how to introduce money creation in an environment with no commitment. A further step could be to incorporate prices in such a framework. The understanding of how money is injected and how non bank agents obtain money may help close the gap between 'pure' monetary theory and applied monetary theory studying monetary policy.

The other literature is that on the theory of banks and financial intermediation. Diamond's (1984) seminal paper on financial intermediation forcefully made the point that financial intermediaries act as delegated monitors. In this paper, the enforcement of contract is delegated to the bank. Discussing the implied similarities and differences of financial intermediaries on the one side and the banking system on the other side could yield interesting insights. Another issue that is worth pursuing is the incorporation of banks into a framework where money is used in exchanges[16].

[16] This could build, for example, on Williamson's (1999) work.

This would not only enrich monetary models, but also shed some new light on the theory of financial intermediation, especially concerning liquidity creation and banking crises.

A Proof of lemma 7.1

A heuristic sketch of the proof will be sufficient. Take the following benchmark: at the beginning of period t, the investors' locations at $t-1$ are publicly revealed. The best strategy to avoid j is then to choose any island $i \neq \sigma_{t-1}(k)$. Entrepreneur j would then avoid k with probability $\theta + (1-\theta)\left(1 - \frac{1}{N}\right)$ at each date (failure occurs when k unfortunatly choose an island where k would be shifted, which conditional on a shift occurs with probability $(N-1) \cdot \frac{1}{N}$).

The information available to k being less informative than this benchmark, we get an upper bound to attainable payoff. Then note that the strategy indicated in the lemma yields this payoff. Moreover, any strategy that departs in t_0 from this straegy would yield a loss in t_0 without any future gain (the upper bound being valid from any period onwards). The best strategy is therefore given by the lemma.□

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