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Economic Growth, Skill-Biased Technical Change and Wage Inequality: A Model and Estimations for the U.S. and Europe

by

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Economic Growth, Skill-Biased Technical Change and Wage Inequality: A Model and Estimations for the U.S. and Europe∗

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Abstract

In recent literature skill-biased technical change has been viewed as a major cause for wage inequality. Some modelling and presentation of stylized facts have been undertaken for U.S. time series data. A preliminary study of wage inequality in a model with knowledge as input in an aggregate production function has been presented by Murphy, Riddell and Romer (1998). Although some important forces determining wage inequality are widely accepted we know little about the quantitative impact of each source and differences across countries. We present a growth model of the Romer type with innovation based technical change and two skill groups where the growth of knowledge, the relative supply of the two skill groups, externalities and substitution effects among the two groups are the driving forces for wage inequality. We undertake estimates for U.S. time series data and contrast those estimates with results from some European countries. In particular, we compare parameter estimations for U.S. and German time series data. The paper concludes that there is less wage inequality across skills in Europe in contrast to the U.S. on the macroeconomic level. But, considering disaggregated data we observe some increases in inequality for Germany, too. Although our model reveals important variables for the explanation of wage inequality there may, however, also be other factors, such as trade unions, which have impacted the wage spread.

Key Words: Economic growth, skill-biased technological change, wage inequality

JEL - Classification: J0, O3.

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1 Introduction

Comparing the labor markets of various OECD-member countries we can, for some countries, observe an increasing wage inequality over time. Although numerous studies on wage inequality exist the sources of such inequality are not completely understood.

In order to shed some light on the driving forces of wage inequality this work concentrates on skill-based wage differentials. The pattern of inequality can be decomposed into three main types of inequality: skill-based wage differentials (called college or wage premium); inequality within groups of the same educational level (within group inequality) and age related wage differentials. In general, skill-based inequality could be treated as an indicator of a high demand for high-skilled relative to low-skilled labor. Beside the supply for the different types of labor one has to ask which forces drive the demand for skilled labor possibly generating wage inequality. Numerous attempts have been made to start with this basic approach and to add other forces for inequality, see, for example, Katz and Murphy (1992), Krugman (1994), Murphy et al. (1998), Katz and Autor (1999), Acemoglu (1998, 2000), Aghion et al. (1999) or Aghion (2001). Although the studies mentioned above are very persuasive either in their empirical or theoretical modelling they almost concentrate on U.S. inequality and their models explain little about factors generating differences across countries. Therefore, the attempt of this study is to compare, in the context of a model, U.S. and European time series data in order to give an answer why U.S. or U.K. inequality has risen while in Germany or France it has obviously not.

Empirically one has observed that for many countries the number of high skilled workers (e.g. an increasing number of college graduates) have sharply increased over time. Applying a simple ‘supply and demand’ scheme one should conclude that their wages should decline. However, for some countries this effect has not been observed. There, the wages for skilled people grew more than for people at a lower educational level. An explanation of the increased inequality is given by the concept of the so-called skill-biased technological change. This argumentation is based on the assumption that technology is complementary to skills. If technology grows the demand for skilled labor increases or an increase in the supply of skills induces faster technology growth which leads to an increasing demand of skilled labor, respectively.
An indicator of skill-biased technological change and the demand for skilled labor is the wage- or college premium; the income of high skilled labor relative to wages for workers at a lower skill level. Acemoglu (1998) argues that a shift in the supply of skilled labor decreases the wage premium (substitution effect) in the short run. Induced technological change shifts the demand curve to the right and leads to an increase in the wage premium (technology effect). The predictions of the model presented by Acemoglu (1998) has been examined empirically by Katz and Murphy (1992) who estimate parameter values for the U.S. close to what Acemoglu’s model predicts. However, for other countries, for example E.U. countries, such as France and Germany, we observe different patterns of wage premia. For France it remains almost constant and for West-Germany the wage premium decreases over time on a macroeconomic level. However, on a disaggregated level (e.g. considering certain industrial sectors) we observe increasing wage inequality for Germany, too (see figure 1).

In this paper, we present a growth model to specify forces generating skill-based wage inequality. Our model is a Romer type growth model which essentially extends the idea of the paper by Murphy, Riddell and Romer (1998). In the paper by Murphy et. al. (1998) the authors assume that technical progress leads to wage differentials between high-skilled and unskilled workers. As technical progress occurs, the relative marginal productivity of different inputs change. Yet, if there is sufficient complementarity between skills and new technologies the demand for more educated employees rises, too, which generates an increase in their wages relative to those of the unskilled workers. Finally, we want to note that in contrast to Krusell, Ohanian and Ríos-Rull (2000) we do not consider substitution between unskilled labor and capital but rather between different types of skill groups. Employing an innovation - based growth model, which allows for different skill levels in production, wage inequality in our model is determined by the stock of technological knowledge, the relative supply of the skill groups, externalities and substitution effects across different skills.

The remainder of this paper is organized as follows. Section two presents some stylized facts on wage inequality and the supply of skilled labor of main OECD-member countries. Sections three and four develop the determinants of the wage premium based upon our innovation-based growth model. Section five presents 1See Acemoglu (1998:1057).
estimation results for the U.S. and German economies. Section six concludes.

## 2 Some Stylized Facts

Before deriving a theoretical model of wage inequality one should consider some stylized facts on inequality. Figure 1 presents the pattern of wage premia for four OECD - member countries Germany, the U.K., France and the U.S. It should be mentioned that, in particular, in the case of Germany and the U.S. different measures of wage inequality are compared. Cross-country data of wage differentials are taken from the OECD Employment Outlook (1993, 1996). There, wage inequality is measured by the ratios of the 10th (D1) and 50th (D5) - percentile to the 90th (D9) percentile wage earners.\(^2\) Taking into account that the 90th - percentile of the income distribution shows the wages are almost earned by high skilled workers whereas the 10th percentile shows the wages for low skilled workers, it can be shown that the median income of different skill groups increases with the level of education.

For the U.S. and Germany we also employ different sources. Long time series for U.S. wage data are taken from the U.S. Bureau of the Census (1998, 2000). For Germany a separate time series of wage inequality is constructed by taking data of the ‘Fachserie 16’ published by the Federal Office of Statistics of Germany. There, we compare wages earned by employees at supervisory job position to wage earners at lower job positions.\(^3\)

Considering figure 1 one observes that the D9/D5 - ratios increase moderately or remain constant for each country. Furthermore, the D9/D1 - ratios, regarding the OECD data, increase sharply for the U.S. and the United Kingdom. For Germany one observes a decreasing pattern while for France the D9/D1-ratio remains roughly constant.

Concentrating on Germany and having a look at the data taken from the Federal Statistical Office we observe increasing inequality for the employees in the West German manufacturing sector. In particular, we observe a sharp increase of the wage premium since the middle of the 1990’s. For the U.S. considering the Census Bureau data (1998) we observe that until the end of the 1970’s the wage premium

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\(^2\)See e.g. Katz and Autor (1999) or Murphy et al. (1998) for similar approaches.

\(^3\)It should be mentioned that we concentrate on West German time series only. Time series for the reunified Germany are not taken into account because of outliers and measurement errors. Furthermore, see Appendix C for our computations.
for college educated workers to non-college education increases slowly. During the 1980’s we observe a sharp increase of this ratio while the increase slows down at the beginning of the 1990’s.

![Figure 1: Patterns of Wage Premia](image)

Figures 2 and 3 show an increasing employment of high skilled labor and a declining fraction of workers without college or university education. In particular, we observe that for both countries the fraction of college educated employees has nearly doubled between 1979 and 1999! Figure 2 presents the ratio of college educated workers (bachelor’s degree and higher) in per cent of total employees (solid line) for the U.S.. Figure 3 shows the ratio of university educated employees to total employees for Germany (solid line). The dashed lines of figures 2 and 3 show the fraction of workers without college or university education. For both countries we observe that
the fraction of ‘low-skilled’ workers declines over time.

Figure 2: High- and Low Skilled Employment, U.S. 1963-1999 (Index 1963=1).
Source: U.S. Bureau of the Census and own calculations

Figure 3: High- and Low Skilled Employment, Germany 1979-1999 (Index 1979=1).
Source: Federal Office of Statistics and own calculations

We can conclude that the above two figures indicate a rising demand for high skilled labor in both countries. This argument could be related, at least in the case of the U.S., to the increasing wage premium. For Germany this argument does not seem to be convincing because we observe, at least on the aggregate, no increase in the wage premium.

In line with Acemoglu (2000) one might interpret figures 1, 2 and 3 that there is a shift in the demand curve for skilled labor (skill-biased technological change) in the U.S. On the other hand for the German economy there seems to be less of an increase in inequality at least as for as indicated by our aggregate measures. The following model attempts to explain the above stylized facts in the context of an innovation-based growth model.

3 The Growth Model

Our model is an extension of the approach by Murphy, Riddel and Romer (1998). Yet, we want to note that they model their idea not in the context of a growth model completely. To achieve this we start with the Romer (1990) growth model and introduce two groups of households. Additionally, we assume that the number

\footnote{See e.g. Katz and Murphy (1992) for a supply and demand explanation of wage inequality.}
of capital goods, i.e. the designs $A$, positively affects the efficiency of both unskilled and skilled labor. That is we assume that knowledge capital is associated with positive externalities. We do this because technical progress is embodied in new capital goods. So, the installment of a new machine does not only raise the capital stock but it also increases the productivity of the labor input. That is any worker is expected to produce more output with the new machine compared to the old machine.

The structure of the productive sectors is the same as in the basic Romer (1990) model. So, we will only briefly sketch the derivation of the equations. To integrate our idea into the Romer (1990) growth model, we first have to introduce a modified Cobb-Douglas production function

$$ Y = K^{1-\alpha} A^\alpha \eta^{\alpha-1} \left\{ \gamma_1 \left[ A^\xi (H - H_A) \right]^{\sigma_p^{-1}} \frac{\sigma_p}{\sigma_p} + (1 - \gamma_1) [A^\epsilon L]^{\sigma_p^{-1}} \right\}^{\alpha \sigma_p / \sigma_p - 1}, \quad (1) $$

with $K$ physical capital, $H_Y$ highly qualified employees producing output and $H_A$ highly qualified employees engaged in R&D. $H = H_Y + H_A$ gives the total number of high skilled workers of the economy. $L$ gives the number of low skilled workers who only produce output. $\sigma_p > 1$, finally, gives the elasticity of substitution between $H_Y$ and $L$. $\xi$ and $\epsilon$ measure the impact of the external effect, i.e. the impact of technical progress, on $H_Y$ and $L$. $\eta$ gives the units of foregone output which are needed to produce one unit of an intermediate good.

Thus, the capital accumulation equation can be written as

$$ \dot{K} = K^{1-\alpha} A^\alpha \eta^{\alpha-1} \left\{ X \right\}^{\alpha \sigma_p / \sigma_p - 1} - C, \quad (2) $$

where

$$ X = \gamma_1 \left[ A^\xi (H - H_A) \right]^{\sigma_p^{-1}} \frac{\sigma_p}{\sigma_p} + (1 - \gamma_1) [A^\epsilon L]^{\sigma_p^{-1}} \right\}. \quad (3) $$

The firms in the final good sector behave competitively. The solution to their optimization problem again gives the inverse demand function for the intermediate good $x(i)$. With the production function (1) it follows

$$ p(i) = X^{\alpha \sigma_p / \sigma_p - 1} (1 - \alpha) K^{-\alpha} \eta^{-\alpha} A^\alpha. \quad (4) $$

The intermediate firm which produces $x(i)$ takes this function as given in solving its optimization problem. The solution to this problem gives the interest rate as

$$ r = (1 - \alpha)^2 \eta^{\alpha-1} K^{-\alpha} A^\alpha X^{\alpha \sigma_p / \sigma_p - 1}. \quad (5) $$
Neglecting depreciation of knowledge, the differential equation describing the evolution of the stock of knowledge or the number of designs, $A$, is given by

$$\dot{A} = \mu H_A^\gamma A^\phi,$$

with $\gamma, \phi \in (0, 1)$.

The differential equation describing the evolution of $H_A$ over time is obtained as follows: First, one uses the fact that the price of knowledge at time $t$, $P_A(t)$, is equal to the present value of the stream of profits, $\pi$, of each intermediate firm because the research sector behaves competitively. This leads to

$$\dot{P}_A = rP_A - \pi,$$

with $\pi = r x \eta \alpha / (1 - \alpha)$. Second, in equilibrium the rental rate of human capital in the final good sector and in the research sector must be equal. This fact gives rise to the following differential equation

$$\frac{\dot{P}_A}{P_A} = \left( \frac{\alpha \sigma_p}{\sigma_p - 1} - 1 \right) \frac{\dot{X}}{X} + \left( 1 - \alpha \right) \frac{\dot{K}}{K} + \left( \frac{\sigma_p - 1}{\sigma_p} - 1 \right) \frac{\dot{H} - \dot{H}_A}{H - H_A} +$$

$$(1 - \gamma) \frac{\dot{H}_A}{H_A} + \left( \alpha - \phi + \xi \frac{\sigma_p - 1}{\sigma_p} \right) \frac{\dot{A}}{A}.$$ (8)

Dividing (7) by $P_A$, setting the resulting expression equal to (8) and solving for $\dot{H}_A$ yields

$$\dot{H}_A = Z^{-1} \cdot$$

$$\left[ \frac{\gamma}{\gamma_1} (1 - \alpha) \mu X H_A^{-\gamma} (H - H_A)^{\frac{1-\gamma}{\gamma}} A^{\phi - 1 + \xi} \frac{\sigma_p - 1}{\sigma_p} - (1 - \alpha) (H - H_A) \frac{C}{K} + \right.$$  

$$\left( \frac{\sigma_p - 1}{\sigma_p} - 1 \right) g_H H + \left( \alpha - \phi + \xi \frac{\sigma_p - 1}{\sigma_p} \right) \mu H_A^\gamma A^{\phi - 1} (H - H_A) + X^{-1} \cdot$$

$$\left( \frac{\alpha \sigma_p}{\sigma_p - 1} - 1 \right) \frac{\sigma_p - 1}{\sigma_p} \gamma_1 (A^\xi (H - H_A)) \frac{\sigma_p - 1}{\sigma_p} \cdot \left( \xi \mu H_A^\gamma A^{\phi - 1} (H - H_A) + H g_H \right) +$$

$$(H - H_A) \alpha (1 - \alpha) \eta^{\alpha - 1} K^{-\alpha} A^\alpha X^{\alpha \sigma_p / (1 - \sigma_p)} +$$

$$X^{-1} \left( \frac{\alpha \sigma_p}{\sigma_p - 1} - 1 \right) \frac{\sigma_p - 1}{\sigma_p} (1 - \gamma_1) (A^{\xi} L) \frac{\sigma_p - 1}{\sigma_p} (H - H_A) \cdot$$

$$\left( \xi \mu H_A^\gamma A^{\phi - 1} + g_L \right),$$ (9)
with \( Z = \left( \frac{a_p - 1}{a_p} \right) - (1 - \gamma) \frac{H - H_A}{H_A} + \gamma (\frac{a_p - 1}{a_p}) X^{-1} \left( \frac{a_p}{a_p - 1} \right) (A^\xi (H - H_A)) \frac{a_p - 1}{a_p} \). 

\( g_H \) and \( g_L \) give the growth rate of the total stock of human capital or skilled labor \( H \) and the growth rate of low skilled labor \( L \), i.e.

\[
\dot{H} = H g_H, \quad (10)
\]

\[
\dot{L} = L g_L. \quad (11)
\]

The model is completed by modelling the household sector. This sector consists of two representative households. The first supplies skilled labor \( H \), the second supplies unskilled labor \( L \). The optimization problem of the household is given by

\[
\max_{C_j} \int_{0}^{\infty} e^{-\rho_j t} u_j(C_j(t))dt, \quad j = H, L. \quad (12)
\]

subject to their budget constraints. Further, the following identities hold, \( K_H + K_L = K \) and \( C_H + C_L = C \), i.e. the total aggregate capital stock and consumption equal the sum of the capital stocks and consumption of both households.

The optimization problem gives the growth rates of consumption as

\[
\frac{\dot{C}_j}{C_j} = -\frac{\rho_j - r}{\sigma_j}, \quad j = H, L, \quad (13)
\]

with \( 1/\sigma_j \) the constant intertemporal elasticity of substitution of consumption between two points in time of household \( j \), and \( \rho_j \) is the subjective rate of time preference of household \( j \), \( j = H, L \).

Our economy is completely described by equations (2), (6), (9), (10), (11) and (14), with the interest rate \( r \) given by (5). In the following we will focus on the wage premium, i.e. the ratio of the wages earned by high-skilled workers to those of low-skilled workers, and derive implications of our model for this variable on the Balanced Growth Path (BGP).

For this model a BGP is derived as for the semi-endogenous growth model with R&D as in Jones (1995). We define a BGP as a path with a constant output/capital ratio, \( Y/K \), and where all variables grow at constant but not necessarily equal rates.

This implies

\[
\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{C}_H}{C_H} = \frac{\dot{C}_L}{C_L}. \]

It should be noted that a common growth rate of consumption of the two households implies that aggregate consumption grows at the same rate. That holds be-
cause of
\[
\frac{\dot{C}}{C} = \frac{\dot{C}_H C_H}{C_H C} + \frac{\dot{C}_L C_L}{C_L C} = g \left( \frac{C_H}{C} + \frac{C_L}{C} \right) = -\frac{\rho_j - r}{\sigma_j}, \quad j = H, L,
\] (14)

with \( g = \dot{C}_H / C_H = \dot{C}_L / C_L \).

It should be mentioned that the intertemporal elasticity of substitution of the two households and the subjective rate of time preference need not necessarily be equal. However, for a common growth rate of consumption to exist those parameters cannot take arbitrary values. That holds because a common growth rate for \( C_H \) and \( C_L \) implies that
\[
\frac{\sigma_L}{\sigma_H} = \frac{\rho_L - r}{\rho_H - r}
\]
has always to be fulfilled.

Further, using \( d/dt (\dot{A}/A) = 0 \) and \( d/dt (\dot{K}/K) = 0 \) yields \( \dot{A} = \frac{\gamma}{1-\phi} g_{H_A}, \quad \dot{K} = \frac{\gamma}{1-\phi} g_{H_A} + \frac{\sigma_p - 1}{\sigma_p} \dot{X} \), where we define \( g_{H_A} = H_A / H_A \).

If the growth rates of \( A \) and \( H_Y \) are constant it can easily be shown that the following relation holds concerning the growth rate of \( X \),
\[
\lim_{t \to \infty} \frac{\dot{X}}{X} = \frac{\sigma_p - 1}{\sigma_p} (\xi g_A + g_{H_Y}), \quad \text{for} \quad (\epsilon g_A + g_L) < (\xi g_A + g_{H_Y}).
\]
The condition \((\epsilon g_A + g_L) < (\xi g_A + g_{H_Y})\) does not pose a sincere limitation of our model. This holds because, on the one hand, the growth rate of high skilled labor in the final goods sector, \( g_{H_Y} \), has been larger than the growth rate of unskilled labor, \( g_L \), in the industrialized countries in the recent decades. Further, on the other hand, it is to be expected that the external effect associated with new machines, i.e. with a rise in \( A \), is higher for skilled labor than for unskilled labor, implying \( \xi > \epsilon \).

Moreover, dividing (7) by \( P_A \) and setting it equal to (8) shows that on a BGP \( H_A \) and \( H_Y \) must grow at the same rate.\(^5\) This fact together with the condition \( H_A + H_Y = H \) implies that \( H \) also grows at the rate with which \( H_A \) and \( H_Y \) grow on the BGP, i.e. we have \( g_{H_A} = g_{H_Y} = g_H \). Thus, on a BGP we have
\[
\frac{\dot{K}}{K} - g_H - g_L = g_H \frac{\gamma}{1-\phi} (1 + \xi) - g_L.
\]
This equation demonstrates that a sufficient and necessary condition for the growth rate of aggregate variables to exceed the growth rate of skilled and unskilled labor
\(^5\)It can be shown that for \((\epsilon g_A + n) > (\xi g_A + g_{H_Y})\) no BGP with constant and positive per capita growth rate exists.
is
\[ g_H \frac{\gamma}{1 - \phi} (1 + \xi) > g_L. \]  (15)

If the growth rate of human capital equals zero this model does not yield sustained per capita growth in the long run, just as the modified Romer model. Then, on the BGP aggregate output, aggregate physical capital and aggregate consumption grow at the same rate as labor input implying that per capita variables are constant. Looking at equation (15) one realizes that this condition is more likely to be fulfilled the more productive human capital in the research process is, i.e. the higher \( \gamma \), and the larger the positive externality of new designs, i.e. the higher \( \xi \).

However, sustained per capita growth only occurs if the growth rate of human capital is positive, that is if \( g_H > 0 \). Thus, we again get a semi-endogenous growth model where conventional government policies cannot affect the long run balanced growth rate. In the next section we will discuss the implications of this model to the wage premium assuming competitive labor markets.

4 The Wage Premium

The wage premium is defined as the ratio of the wages earned by high-skilled workers and the wages earned by low-skilled workers. This variable is of potential interest to economists because it can be seen as a measure of the flexibility of the labor market which has repercussions for the unemployment rate in an economy.\(^6\) Further, the wage premium reflects the inequality between high-skilled and low-skilled workers. If the wage premium rises, the gap between employees getting high wages and employees getting low wages widens which tends to make the income distribution more unequal. If the wage premium falls the reverse holds.

As concerns the determination of the wage premium it is affected by two factors. First, an increase in the supply of high-skilled workers reduces the wage rate for this kind of work and tends to reduce the wage premium. Second, a rise in the number of skilled workers implies that the profitability of technologies increases which are complementary to skilled labor (cf. Acemoglu, 1998). This has a positive effect on the wages of high-skilled workers and, consequently, raises the wage premium. It should be noted that the increase in the supply of high-skilled workers may be the

\(^6\)For a detailed study on the effect of wage inequality on unemployment, see Nickel (1997). Of course, in our model we do not explicitly take into consideration unemployment.
result of government policies. If a government decides to raise the expenditures for education the number of high-skilled workers is expected to rise over time. If it reduces public spending for education, the converse holds.

To derive the wage premium in our model we recall that the production function is given by

\[ Y = K^{1-\alpha} A^\alpha \eta^{\alpha-1} \{X\}^{\frac{\sigma_p - 1}{\sigma_p-1}}, \]  

(16)

with \[ X = \gamma_1 \left[A^\xi(H - H_A)\right]^{\frac{\sigma_p - 1}{\sigma_p}} + (1 - \gamma_1) \left[A^\xi L\right]^{\frac{\sigma_p - 1}{\sigma_p}} \] and \[ H - H_A = H_Y. \] If we assume competitive markets, the wages of high and low qualified employees are equal to the marginal products of high and low qualified workers in the production sector. This gives

\[ w_H = \alpha \gamma_1 \eta^{\alpha-1} K^{1-\alpha} A^\alpha X^{\frac{\sigma_p - 1}{\sigma_p-1}} A^{\xi(\sigma_p - 1)} H_Y^{-\frac{1}{\sigma_p}}, \]  

(17)

\[ w_L = \eta^{\alpha-1} \alpha (1 - \gamma_1) K^{1-\alpha} A^\alpha X^{\frac{\sigma_p - 1}{\sigma_p-1}} A^{\xi(\sigma_p - 1)} L^{-\frac{1}{\sigma_p}}. \]  

(18)

The ratio of the marginal products of the two types of labor, the wage premium \( w_p \), is given by:

\[ w_p = \frac{w_H}{w_L} = \frac{\gamma_1}{1 - \gamma_1} \left[ A^\xi \right]^{\frac{\sigma_p - 1}{\sigma_p}} \left[ H_Y L \right]^{-\frac{1}{\sigma_p}}. \]  

(19)

This result is similar to the one obtained in the paper by Murphy, Riddell and Romer (1998)\(^7\)

\[ \tilde{w}_p = c \left[ A(t) \right]^{\frac{\sigma - 1}{\sigma}} \left[ H \gamma L \right]^{-\frac{1}{\sigma}}, \]  

(20)

where \( c \) denotes a positive constant, \( \sigma \) is the elasticity of substitution, \( A(t) \) and \( B(t) \) are levels of technological knowledge available to high and low skilled workers. The main differences between equations (19) and (20) are that in our model there exists only one stock of technological knowledge which is available to any worker. Furthermore, equation (19) assumes an external effect of technical change (\( \xi - \epsilon \)).

Defining

\[ A_{w_p} \equiv \frac{A^\xi}{A'}, \quad L_{w_p} \equiv \frac{H_Y}{L}, \]  

(21)

we can derive a differential equation describing the evolution of the ratio \( w_p \) which is

\[ \frac{\dot{w}_p}{w_p} = \left( \frac{\sigma_p - 1}{\sigma_p} \right) \frac{\dot{A}_{w_p}}{A_{w_p}} - \left( \frac{1}{\sigma_p} \right) \frac{\dot{L}_{w_p}}{L_{w_p}}. \]  

(22)

\(^7\)See Murphy et.al. (1998:294).
From the definitions of $A_{w_p}$ and $L_{w_p}$ we get
\[
\frac{\dot{A}_{w_p}}{A_{w_p}} = (\xi - \epsilon) \frac{\dot{A}}{A} \quad \text{and} \quad \frac{\dot{L}_{w_p}}{L_{w_p}} = \frac{\dot{H}_Y}{H_Y} - \frac{\dot{L}}{L}.
\] (23)

Considering the wage premium, equation (19), we see that four main factors determine this variable.

First, the quotient of the productivity parameters $\gamma_1/(1 - \gamma_1)$. If $\gamma_1$ is very small and close to zero the wage premium will have a small value, too. A small value for $\gamma_1$ means that the productivity of the high-skilled workers relative to the low-skilled workers is small, i.e. low-skilled workers contribute more to the output than high-skilled workers. Consequently, the wage of the low-skilled workers is relatively high and the wage premium is relatively low. If $\gamma_1$ is large, say near to one, the reverse holds. That is the productivity of the high-skilled workers is relatively high and, as a consequence, their wage rate and the wage premium are high, too.

Second, the ratio $A^\xi / A^\epsilon$ affects the wage premium. A high (low) value for $\xi$ relative to $\epsilon$ means that the positive external effect of technical change affects high-skilled workers to a greater (lower) degree compared to low-skilled workers. That is, technical change, an increase in $A$, leads to a stronger (smaller) increase in the productivity of high-skilled workers compared to low-skilled workers. As a consequence, for a given level of $A$ the wage rate and the wage premium of the high-skilled workers will be relatively high (low).

Third, the number of high-skilled workers relative to the number of low-skilled workers determines the wage premium. If this ratio is high the supply of high-skilled workers is relatively large. As a consequence, the wage premium will take on a low value.

The fourth factor which affects the wage premium is the elasticity of substitution between high-skilled and low-skilled workers, $\sigma_p$. To find the effect of $\sigma_p$ on the wage differential we rewrite (19) and get
\[
w_p = \frac{w_H}{w_L} = \frac{\gamma_1}{1 - \gamma_1} A^{\xi - \epsilon} \left( A^{\epsilon - \xi} \frac{L}{H_Y} \right)^{\frac{1}{\sigma_p}}.
\] (24)

Differentiating (24) with respect to $\sigma_p$ gives
\[
\frac{\partial (w_H/w_L)}{\partial \sigma_p} = - \frac{\gamma_1}{1 - \gamma_1} A^{\xi - \epsilon} \left[ A^{\epsilon - \xi} \frac{L}{H_Y} \right]^{\frac{1}{\sigma_p} - 2} \ln \left[ A^{\epsilon - \xi} \frac{L}{H_Y} \right] \sigma_p^{-2}.
\] (25)
This expression is positive (negative) for $A^{\xi-\epsilon}(L/H_Y) < (>) 1$. This implies that a higher elasticity of substitution raises (reduces) the wage differential if the ratio $(L/H_Y)$ is relatively large (small), i.e. if it is larger (smaller) than the threshold level $A^{\xi-\epsilon}$. That means if the supply of unskilled workers is relatively high an increase in the elasticity of substitution between high-skilled and low-skilled workers raises the wage differential. If the supply of unskilled workers is low a higher elasticity of substitution between high-skilled and low-skilled workers reduces the wage differential.

The growth rate of the wage differential is given by equation (19) together with (22) and (23). It crucially depends on the elasticity of substitution between high-skilled and low-skilled workers, i.e. on $\sigma_p$. The effect of $\sigma_p$ on the growth rate of the wage differential is obtained by differentiating (22) with respect to $\sigma_p$. This gives

$$\frac{\partial(\dot{w}_p/w_p)}{\partial \sigma_p} = \frac{1}{\sigma_p^2} \left[ \frac{\dot{A}_{wp}}{A_{wp}} + \frac{\dot{L}_{wp}}{L_{wp}} \right].$$

If the sum in brackets is positive an increase in the elasticity of substitution raises the growth rate of the wage differential. This means the difference between high-skilled and low-skilled wages rises with a higher elasticity of substitution provided the term in brackets is positive. This latter expression is composed of two parts. First, the difference between the growth rates of the labor productivity of high-skilled and low-skilled workers. A positive difference tends to make the expressions in brackets positive. Second, the difference between the growth rates of high-skilled workers and low-skilled workers. If this expression is positive the term in brackets tends to be positive, too.

Thus, we can state that an increase in the elasticity of substitution between high and low-skilled workers raises the wage differential if the growth rate of productivity of high-skilled workers (as a result of positive externalities of physical capital) is larger than the one of low-skilled workers and if the growth rate of high-skilled labor exceeds the growth rate of low-skilled labor. For example, if high-skilled and low-skilled labor grows at the same rate only the growth rate of the labor productivity of the two groups affects equation (25). Then the growth rate of the wage differential is the higher the higher the elasticity of substitution between high-skilled and low-skilled workers, provided the labor productivity of high-skilled workers grows faster than the labor productivity of low-skilled workers. If the labor productivity of the
low-skilled workers grows faster than the one of the high-skilled the reverse holds. Then, an increase in the elasticity of substitution reduces the growth rate of the wage differential, i.e. the difference between high-skilled and low-skilled wages becomes smaller over time.

Further, it is easily seen that $\xi > \epsilon$ implies that the wage differential rises and vice versa, if skilled and unskilled labor grow at the same rate. This makes sense from the economic point of view, because $\xi > (\leq) \epsilon$ means that the labor productivity of skilled labor grows faster (slower) than that of the unskilled workers. If $\xi = \epsilon$ only the growth rates of skilled and unskilled labor supply affect the growth rate of the wage premium. If skilled labor grows faster (slower) than unskilled labor the wage premium falls (rises) which is reasonable from an economic point of view. Thus, technical progress may drive a wedge between the income of well qualified and less qualified workers making the income distribution more unequal over time. This result is close to the model of Aghion (2001). There, the increase in wage inequality depends crucially on the diffusion of so-called General Purpose Technologies (GPT) and phases of innovation. In a first phase, researchers are less productive than ‘normal’ workers but if the new technology diffuses researchers or high skilled people become more productive than lower skilled workers. In this work, a similar effect is captured by the sign of the difference between $\xi$ and $\epsilon$.

Now, one can identify four influences determining the wage premium:

1. the growth of knowledge, $g_A$
   This variable represents the effects of an economywide stock of knowledge on the wage premium.

2. the growth rate of the supply of the two types of labor, $g_H, g_L$
   This effect indicates how the relative supply of each kind of workers affect the wage premium.

3. the technology effect
   The technology effect is driven by the sign of $(\xi - \epsilon)$. If $\xi > \epsilon$ leads to a higher productivity of high skilled workers. I.e. this leads to a higher demand for skilled workers which increases the wage differential.

4. the elasticity of substitution:
The elasticity of substitution between high and low skilled workers measures the effect how high skilled workers can be replaced by low skilled ones.

Referring to equations (19) and (22) the main parameters of interest are the elasticity of substitution and the technology effect. Knowledge about the sign and values of these parameters allows for a better understanding of the forces driving the different patterns of wage inequality. Taking logs of equation (19) and differentiating with respect to time we obtain the growth rate of the wage premium:

$$\hat{w}_p = \frac{\hat{w}_p}{w_p} = \left(\frac{\sigma_p - 1}{\sigma_p}\right)(\xi - \epsilon)g_A - \frac{1}{\sigma_p}\left(g_H - g_L\right),$$

(27)

where $g_A = \frac{\dot{A}}{A}$, $g_H = \frac{\dot{H}_Y}{H_Y}$ and $g_L = \frac{i}{L}$. Now, equation (27) allows for a closer examination of the technology effect and of the value of the elasticity of substitution. The following section presents time series data and estimations for two OECD member countries, Germany and the U.S.

5 Data and Estimation

5.1 Data Sources and Computations

Considering equation (27) we need data for $H_Y$, $L$ and $A$. $H_Y$ represents employed civilian labor force at a higher educational level for example which have earned a college degree (bachelor’s degree and higher). $L$ denotes the number of employees at a lower educational level. For the U.S. the data are taken from the Annual Statistical Abstract (various issues since 1965) and the U.S. Bureau of the Census (1998, 2000). The German Series are taken from the Federal Statistical Office (1978-2000). The time series of median wages and wage dispersions are taken from the U.S. Bureau of the Census (1998, 2000) and from the ‘Fachserie 16’ published by the Federal Statistical Office of Germany. We have to point out that the applied German data represent only wages and numbers employees of the West German manufacturing sector.

A primary problem is to construct a reliable measure of the stock of knowledge $A$. In particular, various measures of a stock of knowledge exist. A measure of

---

8Note, however, that the manufacturing sector represents in Germany a large fraction of the GDP, roughly 50 percent.

9See e.g. Gong et al. (2001) or OECD (1996), for an appropriate overview.
a stock of knowledge should include innovative investments, a measurable output of knowledge production and the flow of information on knowledge. As an approximation of the first two measures we take R& D - investment and the number of national patent grants. The third item is difficult to approximate. It could include trade flows of technology, the number of internet connections or the number of scientific workshops and conferences. To be consistent with the model of section 4 a closed economy without foreign trade is assumed. Furthermore, taking the growing number of internet connections into account one might assume that the information flow across industrialized countries like U.S. and Germany is the same. Therefore, we apply three different approximations of the ‘growth rate of knowledge’ in our estimations. First, we take the growth rate of total R&D - Expenditures per GDP. Second, we constructed a series by calculating the mean growth rate of the R&D intensity and the growth rate of national patent grants. This measure represents an inventive input (R&D Expenditures) and a possible output of such investments.\textsuperscript{10} Our third method builds on Coe and Helpman (1995) and Gong et al. (2000), who constructed a stock of knowledge by applying the perpetual inventory method.\textsuperscript{11} In this case the stock of knowledge $A$ is approximated through cumulated real R&D - expenditure. The following table summarizes the construction of our time series data:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{w}_p$</td>
<td>Time Series of Wage Differentials</td>
</tr>
<tr>
<td>$g_A$</td>
<td>growth rate of R&amp;D - Intensities</td>
</tr>
<tr>
<td>$g_{A}$</td>
<td>mean growth rate of R&amp;D Intensity and Patents</td>
</tr>
<tr>
<td>$g_{H}$</td>
<td>growth rate of the Stock of Knowledge ($A$)</td>
</tr>
<tr>
<td>$g_{H}$</td>
<td>Employees with college or higher education</td>
</tr>
<tr>
<td>$g_{L}$</td>
<td>Employees without college or higher education</td>
</tr>
</tbody>
</table>

Before concentrating on the parameter estimations we present some main properties of our constructed time series data. For the U.S. we concentrate on two different

\textsuperscript{10}See Siegel (1999) for a brief survey of similar approaches of approximating the stock of knowledge.

\textsuperscript{11}See Appendix B for a brief description of the perpetual inventory method.
kinds of wage premia. First, we calculate the wage premium of employees with college education to employees without college education. This time series is called ‘wage premium’. Secondly, we calculate a series where we compare the wages of employees with some college degree over employees with high school education. According to Murphy et. al. (1998) we call this series ‘college premium’. Table 2 shows some characteristics of the applied U.S. data. It should be noted that, according to table 1 and eqn. (27), each variable is measured in growth rates.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage Premium</td>
<td>0.0078</td>
<td>0.0258</td>
</tr>
<tr>
<td>College Premium</td>
<td>0.0089</td>
<td>0.0226</td>
</tr>
<tr>
<td>bach. degree</td>
<td>0.0400</td>
<td>0.0377</td>
</tr>
<tr>
<td>no bach.</td>
<td>0.0215</td>
<td>0.0277</td>
</tr>
<tr>
<td>high school</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R&amp;D - Intensity</td>
<td>-0.0005</td>
<td>0.0306</td>
</tr>
<tr>
<td>R&amp;D + Patents</td>
<td>0.0110</td>
<td>0.0360</td>
</tr>
<tr>
<td>Stock of Knowl. (A)</td>
<td>0.0262</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

For the U.S. we observe a small mean growth rate of the wage and college premia. Furthermore, the growth rate of technological change seems to be small, too. The mean growth rate of $A$ and the ratios of high-skilled labor to the labor with lower skills (see line 4 and 5) are positive too.

Comparing the results for the U.S. with the German data (see table 3) we observe that the mean growth rate of wage inequality is in the U.S. four times higher than in Germany. Furthermore, table 3 shows that the mean growth rate of the R&D - Intensity is low, in the case of the U.S. it is negative. Yet, for both countries we observe growth rates of $A$ significantly positive. It is interesting to note that for most of the variables the U.S. time series exhibit a higher standard deviation. This, in particular holds for the wage premium, apparently reflecting the well known differences in the mobility of labor of the two countries.
Table 3: Time Series Properties, German Data (1974-98)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage Premium</td>
<td>0.0019</td>
<td>0.0068</td>
</tr>
<tr>
<td>Higher Educ.</td>
<td>0.0111</td>
<td>0.0231</td>
</tr>
<tr>
<td>Lower Educ.</td>
<td>0.0111</td>
<td>0.0231</td>
</tr>
<tr>
<td>R&amp;D Intensity</td>
<td>0.0028</td>
<td>0.0365</td>
</tr>
<tr>
<td>R&amp;D + Patents</td>
<td>0.0098</td>
<td>0.0192</td>
</tr>
<tr>
<td>Stock of Knowl. A</td>
<td>0.0431</td>
<td>0.0139</td>
</tr>
</tbody>
</table>

5.2 Estimation Results

Before we estimate the parameters of our wage premium model (28), we want to examine simple correlations between the wage premium and the main variables such as the stock of knowledge and the relative supply of skilled workers. We employ simple OLS regressions for the wage premium and the other variables. The results are illustrated in appendix A and summarized in table 4. As one would expect most of our proxies of knowledge show a positive influence of knowledge on the wage premium. This seems to hold for the U.S. as well as for Germany. Results of the literature on skill-biased technological change seem to be supported by the results of table 4. On the other hand whereas the relative increase of high skilled labor, compared to low skilled labor, does not make the wage premium for the U.S. falling - but rather rising - for Germany one can observe the opposite result.

Table 4: Sign of Correlations

<table>
<thead>
<tr>
<th>Country</th>
<th>Variable</th>
<th>R&amp;D Intensity</th>
<th>R&amp;D + Patents</th>
<th>gA</th>
<th>gH - gL</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Wage Premium</td>
<td></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>U.S. College Premium</td>
<td></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>Germany Wage Prem.</td>
<td></td>
<td>+</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The results of table 4 indicates positive technology effects and, except for the U.S., negative supply effects. Therefore, we conclude that in the U.S. skilled workers are more scarce compared to unskilled workers which gives rise to an increasing wage premium than in Germany.
Next we estimate the parameters of the wage premium, given by equation (27). Because of the structure of equation (27) it seems sufficient to apply OLS estimation in a first step. Consistent with the equation (27) and the work of Katz and Murphy (1992) and Murphy, Riddell and Romer (1998) the following regression equation is estimated\(^{12}\)

\[
\hat{w}_p = \beta_1 + \beta_2 g_A - \frac{1}{\beta_3} g_{HL} + \epsilon
\]  

(28)

where \(g_A = \frac{4}{A}\) and \(g_{HL} = (g_H - g_L)\). Mention that \(\beta_1\) represents an arbitrary constant which captures other variables influencing the wage premium (e.g. the role of trade unions). The parameters \(\beta_2\) and \(\beta_3\) represent the so called technology effect and the elasticity of substitution.\(^{13}\) In the first step, not reported here, we have employed OLS regressions. For almost all of the regressions for both the U.S. as well as Germany we obtained the expected sign but the parameters were insignificant. As one can observe from the plots of appendix A the time series are characterized by strong outliers. We, therefore, undertook regressions by a procedure that gives less weight to outliers. We employed a nonlinear estimation technique that gives higher weight for values close to the mean and a lower weight for values far from it. We obtained the following results for the U.S.:

Table 5: Estimation of Eq (28), U.S. (1964-99)

<table>
<thead>
<tr>
<th>Proxy for Knowl.</th>
<th>U.S. Wage Premium</th>
<th>U.S. College Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\beta_1) (\beta_2) (\beta_3)</td>
<td>(\beta_1) (\beta_2) (\beta_3)</td>
</tr>
<tr>
<td>R&amp;D - Intens.</td>
<td>0.0207 0.0720 2.7716</td>
<td>0.0166 0.2959 2.6621</td>
</tr>
<tr>
<td>(std. \ error)</td>
<td>0.0069 0.1586 0.9879</td>
<td>0.0574 0.1375 1.3149</td>
</tr>
<tr>
<td>R&amp;D + Patents</td>
<td>0.0212 0.0066 2.7229</td>
<td>0.0169 0.1645 2.6602</td>
</tr>
<tr>
<td>(std. \ error)</td>
<td>0.0071 0.1330 0.9412</td>
<td>0.0062 0.1256 1.4131</td>
</tr>
<tr>
<td>(A)</td>
<td>0.0177 0.1278 2.7506</td>
<td>0.0118 0.2226 2.8822</td>
</tr>
<tr>
<td>(std. \ error)</td>
<td>0.0177 0.5936 0.9698</td>
<td>0.0189 0.6125 1.8292</td>
</tr>
</tbody>
</table>

\(^{12}\)For our OLS - estimations we employed the following equation:

\[
\hat{w}_p = \beta_1 + \beta_2 g_A + \beta_3 g_{HL} + \epsilon
\]

\(^{13}\)Note that \(\beta_2\) represents:

\[
\beta_2 = \frac{\sigma_p - 1}{\sigma_p} (\xi - \epsilon) \Rightarrow (\xi - \epsilon) \approx \frac{\beta_3}{(\beta_3 - 1)} \beta_2
\]
As one can observe the regression coefficients have the expected sign and most of them are significant. The results for the German economy are presented in table 6.

Table 6: Nonlinear Estimation of Eq (28), Germany (1974-98)

<table>
<thead>
<tr>
<th>Approx. of Knowl.</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D - Intens.</td>
<td>0.0097</td>
<td>0.1271</td>
<td>1.7536</td>
</tr>
<tr>
<td>std. error</td>
<td>0.0019</td>
<td>0.0475</td>
<td>0.2505</td>
</tr>
<tr>
<td>R&amp;D + Patents</td>
<td>0.0122</td>
<td>-0.1139</td>
<td>1.6965</td>
</tr>
<tr>
<td>std. error</td>
<td>0.0024</td>
<td>0.0595</td>
<td>0.2586</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0067</td>
<td>0.2165</td>
<td>1.9261</td>
</tr>
<tr>
<td>std. error</td>
<td>0.0022</td>
<td>0.0940</td>
<td>0.3143</td>
</tr>
</tbody>
</table>

Here, the parameters have almost the expected sign and are significant. The main difference between the results for the U.S. and Germany is that for Germany the elasticity of substitution is much lower than for the U.S. which has dampening effects on the wage premium.

In both countries the knowledge variable has a positive effect and a larger growth rate of skilled compared to unskilled labor has a negative effect on the wage premium. On the other hand the elasticity of substitution between the two types of labor also can dampen the effect of the high growth rate of the supply of skilled labor on the wage premium. Yet, the higher the elasticity of substitution between the two types of labor the less effective is its dampening effect on the wage premium. In other words if there had been a larger growth rates of unskilled labor and a low elasticity of substitution then this would have strongly skewed the income distribution towards skilled labor. Those results depend, however, also on the sign of of the first term in eqn (27). A higher elasticity of substitution reduces this effect.14

6 Conclusion

In the paper we have shown that there are four main factors driving wage inequality. These are the rate of technological change, the technological spill-over effect, the rel-

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14 This is consistent with results in resource economics where the scarce resource obtains overtime the larger income share the smaller the elasticity of substitution between inputs is, see Scholl and Semmler (2000).
ative supply of skilled and unskilled labor and the elasticity of substitution between high and low skilled workers. If technical change, the technological spill-over effect and the growth rate of the ratio of skilled to unskilled labor are given and positive than the elasticity of substitution may mitigate wage inequality. With low elasticity of substitution (below one) wage inequality may strongly fall, if the growth rate of the ratio of skilled to unskilled labor is positive. With high elasticity the wage inequality is less corrected. Thus, the increase in wage inequality is strongly mitigated if the elasticity of substitution is low and the growth rate of skilled compared to unskilled labor is higher. Yet, the inequality increases if the growth rate of low skilled labor is higher than for skilled labor.

Both our model and our estimates for the U.S. time series data allow a coherent interpretation of the trends in wage inequality. In the case of the U.S. the elasticity of substitution is greater than one but lower than for Germany. For Germany, the stylized facts show that there is less wage inequality and if there is, one can observe that the reduction of inequality arising from a positive growth rate of skilled to unskilled labor is mitigated to a lesser extent by the elasticity of substitution. Yet, we want to point out, that in Germany there appear to be also other forces, for example, the influence of the trade union wage setting policies, that have reduced wage inequality over time. We have not taken into account those forces which may also explain part of the trend in the wage premium, particularly for Germany. As the stylized facts in section 2 showed it is only in recent times that wage inequality tends to rise in Germany.

So far we have only discussed inequality across skill groups. Within group inequality describes the observation that wages earned by workers with the same qualification are not the same. Aghion et al. (1999), for example, apply a model with vintage capital and learning by doing effects. There, it is assumed that new capital goods have positive effects on the productivity of workers. The workers can either improve their knowledge through learning activities or remain at the same job. As a result workers become more heterogeneous which leads to increasing wage differentials within groups of similar skill levels.

Another important influence for increasing wage inequality is assumed to arise from international trade. In particular, if an industrialized country increases its exports of skill-intensive goods and raises its imports of labor intensive goods, the production will shift to skill intensive goods which raises the skill-based wage in-
equality. In the long-run the rising wage inequality will lead to a reduction of the ratio of high skilled to low skilled workers. However, Krugman (1994) argues that if trade liberalization is the main force behind growing wage inequality this would lead to two observable facts: first a declining ratio of skilled to unskilled employment and, secondly, a substantial shift of employment towards skill intensive industries. Krugman (1994) argues further, that both propositions fail to hold and that wage differentials and the relative demand for skilled people has increased because of some “common factors that affect all sectors”.  

Although almost aggregated data are employed in our study, some main influences on the observed patterns of wage inequality could be quantified. Further work with a more precise measure of the stock of technological knowledge, with disaggregated data sets or more precise assumption about the wage setting behavior is left for future research.

\[15\text{See Krugman (1994:36).}\]
Appendix A: Simple Correlations

Figure 4: Correlation US - College Premium (1964-99)
Figure 5: Correlation US - Wage Premium (1964-99)\textsuperscript{16}

\textsuperscript{16}Note that the ‘largest’ outliers of the data of figure 5 (down right) are truncated. Including those observations we would observe no correlation.
Appendix B: Application of the perpetual inventory method

The perpetual inventory method:

$$A_t = (1 - \delta_A)A_{t-1} + R_{t-1}$$
where the initial condition follows as:

\[ A_0 = \frac{R_0}{g + \delta_A}. \]

Note that \( \delta_A \) denotes the depreciation of knowledge and \( g \) is treated as the mean growth rate of R&D expenditures in the whole period. Table 7 shows the parameter values and time period used for calculating the \( A \):

<table>
<thead>
<tr>
<th>Country</th>
<th>( \delta_A )</th>
<th>( g )</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.08</td>
<td>0.028</td>
<td>1960 – 1998</td>
</tr>
<tr>
<td>Germany</td>
<td>0.08</td>
<td>0.050</td>
<td>1963 – 1998</td>
</tr>
<tr>
<td>Coe / Helpman (1995)</td>
<td>0.05 / 0.15</td>
<td>0.050</td>
<td>1963 – 1990 (Germany)</td>
</tr>
<tr>
<td>Gong et. al. (2000)</td>
<td>0.08</td>
<td>0.025</td>
<td>1962 – 1991 (U.S.)</td>
</tr>
</tbody>
</table>

Note that the last four rows show the values of the referred articles.

**Appendix C: Data Computations**

1. **U.S. Data:**

For the U.S., the number of higher educated workers consist of the sum of male and female employees which earned some college degree (bachelor’s degree and higher). Low educated employees consist of the sum of employees which earned a high school degree or less. In particular, the U.S. Bureau of the Census distinguishes between less than the 9th degree, the 9th to 10th degree and the high school degree.

Calculating the wage premia, the wage of high skilled labor is approximated by the median of wages of male and female employees with college education. The wage of low skilled labor is given by the median of wages earned by low educated male and female employees.
In the U.S. case we distinguish between so-called college- and wage-premia:

\[
\text{Wage Premium: } = \frac{\text{Median wages college education}}{\text{median wages Non-College education}}
\]

\[
\text{College Premium: } = \frac{\text{Median wages college education}}{\text{median wages High-School education}}
\]


2. **German Data:**

In contrast to the U.S., for Germany we employ data for the manufacturing sector, only. The available data are for both male and female blue and white collar workers at different job positions. Because of the German education system we assume blue and white collar workers at supervisory job positions as high skilled labor. In particular, such workers are educated at school for at least 12 years, furthermore they normally got a practical education (apprenticeship). In this line, we assume employees at lower job positions and, therefore, at a lower educational level as low skilled labor. The numbers of employees and the wages for the German economy are calculated analogue to the U.S.

\[
\text{Wage Premium: } = \frac{\text{Median wages at supervisory job positions}}{\text{Median wages at lower job positions}}
\]


**Appendix D: Data Sources**


References


