The New Open Economy Macroeconomics of Government Debt

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Abstract

In this paper we introduce an overlapping generations structure of the Blanchard type in a basic New Open Economy Macroeconomics model. This allows us to make a step forward in studying the effects of government debt in the new paradigm. We analyse the effects of a temporary reduction in taxes. Our results suggest that this policy increases the relative consumption of the country that expands. The effects on relative output and the exchange rate depend on the relative weights of leisure and consumption private utility. To the best of the author’s knowledge, this paper is the first contribution in the literature to analyse the effects of government debt in the New Open Economy Macroeconomics framework.

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1 Introduction

Following the path-breaking contribution of Obstfeld and Rogo® (hereafter OR, 1995, 1996), a new paradigm, explicitly based on microfoundations and incorporating market imperfections such as monopolistic competition and price-stickiness, is emerging in open economy macroeconomics. This new literature is often referred to as the New Open Economy Macroeconomics. One of the ambitions of this new class of models is to replace the ad hoc Mundell-Fleming (MF) framework for the analysis of macroeconomics (monetary and fiscal) interdependence. In spite of this, most of the contributions in this growing literature tend to focus on monetary policies, neglecting the effects of fiscal shocks. Furthermore, the few papers that analyse fiscal policy do this in a setup in which Ricardian equivalence holds, and therefore they only consider balanced-budget policies. In this paper we introduce an overlapping generations structure of the Blanchard (1985) type in the basic OR framework. This allows us to make a step forward in studying an issue that we regard as paramount, namely the analysis of the effects of government debt in the New Open Economy Macroeconomics framework. The analysis of the effects of government debt is interesting both for theorists and for policy makers, especially if the new paradigm is ultimately to replace the old MF one in policy oriented analysis. OR themselves stress the importance of breaking down Ricardian equivalence in their framework: "Introducing overlapping generations in place of homogeneous ininitely lived agents would enrich the dynamics while permitting real effects of government budget deficits" (OR 1995, pag.654). This direction of research seems therefore worth pursuing. The theoretical framework that we develop can be useful to give some insights about how government debt affects the macroeconomic interdependence between large economies (such as US-Japan or US-EU relationships).

In this paper we study the international effects of a temporary reduction in taxes, financed by government debt. Our results suggest that, following

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\[1\] For comprehensive surveys, see Lane (2001) and Sarno (2001). Most of the work in progress in this area can be accessed from Bryan Doyle's web page at http://www.geocities.com/brian_m_doyle/open.html.

\[2\] Mundell (1968), Fleming (1962).

\[3\] Two exceptions are Caselli (2001) and Ganelli (2000).

\[4\] An exception is Ghironi (2000), that combines the OR framework with ininitely-lived overlapping generations of the Weil (1989) type. Our contribution differs from his in the use of a different overlapping generations structure and, more importantly, in the fact that he does not analyzes public debt issues.

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this policy, the country that implements it benefits more than the foreign country in terms of increased consumption. We also show that the effect on the exchange rate, and therefore on output, depend on the relative weights of consumption and leisure in the utility function. The latter result confirms that, how stressed by Sarno (2001): "...the implications of such models depend on the specification of preferences." (Sarno 2001, pag. 27). For a parametrization that was considered the sensible one in previous papers\(^5\) (i.e. a weight on leisure lower than the weight on consumption), this policy is likely to depreciate the nominal exchange rate and to increase relative output. The opposite result emerges if the weight on leisure is higher than the one on consumption in the utility function. We also discuss, without explicitly deriving reduced forms, the effects on the levels of the variables. The structure of the paper is the following: next section introduces the model, section 3 derives and discusses the macroeconomic effects of government debt and section 4 concludes.

2 The Model

This is a two country model. In each period \(n\) individuals are born in the Home country and \(1 - n\) in the Foreign country. We assume that the constant probability of surviving to the next period, \(q\), is the same across countries. Home population is therefore \(n^1 = \frac{1}{1 - q} n\) and accordingly, Foreign population is \(\frac{1}{1 - q} n\) and world population is \(\frac{1}{1 - q}\). There is also a measure of (ininitely lived) firms in the world, \(n\) of these are located in the domestic country, \(1 - n\) in the foreign country. Firms have monopolistic power in the production of a differentiated good. Both the firms and the goods that they produce will be indexed by \(z \in [0; 1]\):

Consumption of good \(z\) by domestic and foreign agents of age \(a\) is given by:

\[
C^a_{a,t} = \int_0^1 c^a(z) \mu^{a-1} dz \frac{1}{\mu^{a} - 1}
\]

\[
C^a_{a,t} = \int_0^1 d^a(z) \mu^{a-1} dz \frac{1}{\mu^{a} - 1}
\]

\(^5\)Ganelli (2001a), Ganelli (2001b).
where $\mu$ is the elasticity of substitution between any pair of goods. The corresponding price indexes are:

$$P = \frac{1}{(1-\mu)} \left[ \int_0^Z p(z)^{(1-\mu)} \, dz \right]^{1/(1-\mu)}$$

(1)

$$P^\pi = \frac{1}{(1-\mu)} \left[ \int_0^Z p^\pi(z)^{(1-\mu)} \, dz \right]^{1/(1-\mu)}$$

(2)

where $p(z)$ and $p^\pi(z)$ are respectively the home-currency and foreign-currency price of good $z$:

### 2.1 Production

If we denote the good produced by a generic firm with the index $z$, we can derive the total (private plus public) demand for it as follows:

$$Y_t(z) = \left[ \frac{P(z)}{P} \right]^{1/(1-\mu)} \left[ \sum_{a=0}^n q^a C_{a:t} + q^a G_t \right] + \left[ \frac{P^\pi(z)}{P^\pi} \right]^{1/(1-\mu)} \left[ \sum_{a=0}^n q^a C_{a:t} + (1-n) \sum_{a=0}^n q^a G^\pi_t \right]$$

(3)

Where $p(z)$ and $p^\pi(z)$ are the domestic and foreign currency prices of the representative good $z$; $P$ and $P^\pi$ are the domestic and foreign prices of the composite good and $C^W + G^W$ is world aggregate consumption of it. $G_t$ is to be interpreted as home government spending per-capita, and an analogous interpretation holds for $G^\pi_t$. In deriving the latter expression we have also made use of the law of one price and of purchasing power parity, which implies $P(z) = p^\pi(z)$: $Y_t(z)$ is the demand for output per-firm. Since the $n$ domestic firms behave symmetrically, aggregate output for the Home country is given by $nY_t(z)$: Dividing the latter by the size of the domestic population we get per-capita Home output as $Y^{PC} = Y_t(z)(1-n)$: Equation (3) then implies:

$$Y_{tPC} = \left[ \frac{P(z)}{P} \right]^{1/(1-\mu)} \left[ C^{WD} + G^{WD} \right] (4)$$

Since this variable is distributed independently of age, individual and per-capita coincide for it.
We assume that domestic firms only hire domestic agents and foreign firms only hire foreign agents. Furthermore, the only production factor is labour (with constant returns). The profit-maximising conditions for domestic and foreign firms are therefore the familiar expressions:

\[ W_t = \frac{\mu i}{\mu} p_t(z) \]

and

\[ W^*_t = \frac{\mu i}{\mu} p^*_t(z) \]

where \( W_t \) and \( W^*_t \) are domestic and foreign nominal wages.

2.2 Private Agents

The representative domestic agent maximizes the expected utility function:

\[
E(U_t) = \sum_{s=t}^{\infty} (1-q)^s \left[ \log(C_{a+s_i t; s}) + \bar{A} \log \frac{M_{a+s_i t; s}}{P_s} + \bar{A} \log(1 - L_{a+s_i t; s}) \right]
\]

Where \( \bar{\mu} \) is the discount factor, \( q \) the probability of surviving to the next period, \( C; M = P \) and \( L \) consumption, real balances and labour supply. Preferences are homothetic and separable in consumption, real balances and leisure. The endowment of time in each period is normalized to 1: \( L_{a+s_i t; s} \) is the quantity of labour supplied in every period, \( (1 - L_{a+s_i t; s}) \) is leisure. A standard assumption in this framework is the existence of insurance companies. We assume that insurance companies pay a net premium of \( \frac{1-q}{q} \) on the agent's financial wealth for each period in which the agent is alive, while they encash the agent's financial wealth if the agent dies. Domestic agents can hold wealth either in the form of a credit towards...
private foreign agents or in the form of government debt. In order to make keep notation as simple as possible, we assume that all assets are defined in terms of the composite consumption good, and that agents hold the amount of assets that maximizes their expected utility but they are indifferent to the composition of assets. The representative agent’s period t budget constraint in real terms is, therefore:

\[ F_{a,t+1} + \frac{M_{a,t}}{P_t} + C_{a,t} = \frac{1}{q} \left[ \frac{M_{a,t} \cdot \frac{1}{t+1}}{P_t} \right] + (1 + r_t) F_{a,t+1} + (1 + \gamma_t) \left( \frac{W_{a,t}}{P_t} \cdot L_{a,t} \cdot \frac{1}{P_t} \right) + \delta_t \]

where \( \frac{\delta_t}{P_t} = \frac{1}{q}(z)(1 - q) \) is the per-capita quota of domestic profits\(^9\), \( F \) denotes total assets holdings of the agent and \( r \) the real interest rate. The first order conditions with respect to consumption, leisure and money holdings yield the following expressions\(^{10}\):

\[ C_{a,t} = \left( \frac{1}{1 + \bar{A} + A} \right) f(1 + r_t) \left( \frac{1}{q} \right) \left[ \frac{1}{1 + i_t} \cdot \frac{M_{a,t} \cdot \frac{1}{t+1}}{P_{t+1}} \right] + F_{a,t+1} \] + \( H_{a,t} \cdot g \]

\[ L_{a,t} = 1 \cdot \bar{P}_t \cdot C_{a,t} \]

\[ \frac{M_{a,t}}{P_t} = \bar{A} \left( \frac{1}{i_{t+1}} \right) C_{a,t} \]

Where human wealth is equal to \( H_{a,t} = P \sum_{s=1}^{\infty} \bar{Q}_t \cdot q^s \left( \frac{W_{a,s} \cdot \frac{1}{s}}{P_s} \right) + \frac{1}{P_s} \cdot \delta_s \). Human wealth is defined as the present discounted value of potential gross earnings (that would be earned if the agent chose to consume no leisure),

\(^9\)We assume that profits of domestic \( \bar{A} \) rms are equally distributed across domestic agents (and vice versa). Note that, with \( n \) symmetric \( \bar{A} \) rms in the country, aggregate profits are given by \( n^{1/4}(z) \). Dividing the latter by the size of the country’s population \( \frac{n}{1 + n} \), we get \( \frac{1}{q} (1 - q) \).

\(^{10}\)The derivations follow the same steps of the closed economy model presented in Ganelli (2001a), and are available from the author upon request.
minus taxes. The first order conditions can be aggregated across ages and then divided by the population size in order to get per-capita variables. Following a method analogous to the one used in Ganelli (2001a), we can also derive a law of motion for consumption in per-capita terms. This leaves us with the following set of equations in per-capita terms:

\begin{align}
C_{t}^{PC} &= \left(\frac{1 + q^{-1} + \beta}{1 + \lambda + \lambda}\right)TW_{t}^{PC} \tag{7} \\
\frac{M_{t}^{PC}}{P_{t}} &= A\left(1 + \frac{1 + i_{t+1}}{i_{t+1}}\right)C_{t}^{PC} \tag{8} \\
L_{t}^{PC} &= 1 + \tilde{A}\frac{P_{t}}{\tilde{p}(z)}\frac{\mu}{\lambda}C_{t}^{PC} \tag{9} \\
C_{t+1}^{PC} &= \left(\frac{1 + q^{-1}}{1 + \lambda + \lambda}\right) \left(1 + \frac{q}{H_{t+1}}\right) + (1 + r_{t+1})q^{-1}C_{t}^{PC} \tag{10}
\end{align}

Where \(TW_{t}^{PC}\) is total (financial plus human) per-capita wealth.

The government spends on public expenditure that does not affect private utility. Government expenditure can be financed by seigniorage, lump-sum taxes and debt, according to the single-period budget constraint:

\[G_{t} + (1 + r_{t})D_{t} = \dot{i}_{t} + \left(\frac{M_{t}}{P_{t}} - M_{t+1}\right) + D_{t+1} \tag{11}\]

Integrating the agents' private budget constraint across ages, and substituting for \(\dot{i}_{t}\) from the government budget constraint, we get the following expression in aggregate terms:

\[F_{t+1} = \frac{\mu}{\lambda}p(h)L_{t} + r_{t}F_{t} + C_{t}G_{t} + D_{t+1} + \frac{1}{P_{t}}\tag{12}\]

where \(p(h)\) is the price set by a typical home firm.

\[\text{Of course, as leisure provides utility, agents will not choose to supply a quantity } 1 \text{ of work in each period. This is evident from the first order conditions.}\]

\[\text{Notice that in this equation we have replaced the nominal wage with its value implied by the profit maximization condition.}\]
Defining net foreign assets as \( \text{NFA} = F - D \); and dividing both sides by the size of the domestic population, we get a current account equation in per-capita terms as follows:

\[
\text{NFA}_{t+1}^{\text{PC}} - \text{NFA}_t^{\text{PC}} = \frac{\mu}{\mu_p} \frac{1_p(h)}{P_t} L_t^{\text{PC}} \ i \ C_t^{\text{PC}} + G_t^{\text{PC}} + \frac{r_t}{P_t} \text{NFA}_t^{\text{PC}}
\]

The concept of net foreign assets is similar to the one introduced by OR (1995, 1996). Here, however, the term "net" means not only the net position towards the other country, but also the fact that this is net of assets issued by the domestic government. In the aggregate the following must hold at any time: \( \text{NFA} = \text{i} \text{NFA}^\text{a}; \) This implies a relationship in per capita terms, that will be useful in what follows:

\[
n \text{NFA}_t^{\text{PC}} = \text{i} (1 - n) \text{NFA}^\text{apc}.
\]

We assume that parameters do not vary across countries. Therefore, a set of equations that are the equivalent of (4), (7), (8), (9), (10) and (12) hold for the foreign country, together with an analogous budget constraint for the foreign government.

The main aim of this paper is to assess the macroeconomic impact of a temporary reduction in taxes financed by government debt, in an open economy framework. In order to do this, we need to resort to log-linearization. In what follows we define a convenient initial steady-state around which we will log-linearize the model.

2.3 The Initial Steady State

As in OR (1995, 1996) and in Ganelli (2000), we consider an initial steady state in which net foreign assets, government spending and government debt are all zero. We accordingly denote this steady state using the subscript \( 0 \) : \( \text{NFA}_0 = \text{i} \text{NFA}^\text{a}_0 = 0; D_0 = 0; D^\text{a}_0 = 0; G_0 = 0; G^\text{a}_0 = 0; \) Perfect initial symmetry also implies that \( \frac{\theta(2)}{P_0} = \frac{\theta^a(2)}{P_0} = 1; \) In a steady state defined in this way, the initial per-capita values of output and consumption take, in both countries, the same value. This can be easily shown formally. Considering the steady-state version of the current account equation (12), using the fact that steady state profits are equal to \( \frac{1}{\mu} Y_0^{\text{PC}} \); implies \( Y_0^{\text{PC}} = C_0^{\text{PC}} \); Using this
result in equation (9) together with the assumption that labour is the only factor of production with constant returns, implies the following steady state value:

\[ C^0_{PC} = Y^0_{PC} = \frac{\mu^{-1}}{\mu^{-1} + A} \]  

(13)

Using the equivalent foreign equations we can show that the above value is the one taken also by foreign consumption and output at the initial steady state.

The latter result implies that the initial steady-state level of human wealth is, for both countries:

\[ H_0 = H_0^w = f \frac{\mu^i}{\mu} + \frac{1}{\mu^{-1} + A} g R_0 i q \]

In order to find an expression for the initial real interest rate, we can substitute the above expression, together with the steady state value for consumption, in to the law of motion of consumption (10). The terms in \( \mu \) cancel out, and we find that the solution for the initial gross real interest rate is the same as in Ganelli (2001a), for the case of no initial debt. This is implicitly given by the equation:

\[ R^2_0 = f q + \frac{1}{q^i}[1 + i_i q^{-1} - 1 + A + A^i] q(1 + A) g R_0 i q \]

(14)

The solution for the real interest rate, therefore, is not affected by the introduction of imperfect competition. The real interest rate is the intertemporal price of a consumption aggregate, that does not depend on the elasticity of substitution between the differentiated goods that are aggregated in it.

To solve explicitly for \( R_0 \) from this equation would be possible, but not very illuminating. The implications of (14) are more easily understood looking at Figure 1.

The two solutions of (14) are the points in which the parabola \( f(R) = R^2 \) meets the line \( g(R) = f q + \frac{1}{q^i}[1 + i_i q^{-1} - 1 + A + A^i] q(1 + A) g R_0 i q \). Given the ranges of values of the parameters, the slope of this line is obviously positive.

From Figure 1 it is clear that we are faced with two possible equilibria.

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14Our choice of the initial steady state implies \( z_0 = 0 \):
One problem, therefore, is how to discriminate between them. We can say something about the two different steady-states if we assume that deviations from the Ricardian equivalence case ($q = 1$), are not too large. The first argument is a type of Samuelson correspondence principle: the equilibrium that has desirable stability properties must also yield desirable comparative static properties. In this case, it is easy to check that, as the derivative of $f(q + 1) \left( 1 - \frac{1}{1 + \alpha + \lambda} \right)$, being equal to $-1 \left( \frac{1}{1 + \alpha + \lambda} \right)$, is negative, in the neighborhoods of this value a fall in $q$ will imply an increase of the "higher" equilibrium (that is a movement from $R_H$ to $R_H'$ in Figure 1) and a fall of the "lower" equilibrium (from $R_L$ to $R_L'$ in Figure 1). It follows that, if the deviation from the infinite life case is not too big (if $q$ is not too much smaller than 1), $R_H$ displays the more sensible result in terms of comparative static: when the probability of surviving to the next period becomes smaller, agents become more short sighted and therefore the real interest rate increases (present consumption becomes more costly in terms of future consumption). This would suggest to restrict our attention to the higher equilibrium $R_H$. Another argument that leads to the same
conclusion can be developed considering that, when agents have infinite lives, our model collapses to a discrete time version of the Ramsey (1928) model. When \( q = 1; \) equation (14) yields the two solutions \( \bar{R}_L = 1 \) and \( \bar{R}_H = \frac{1}{2}; \) Since \( \bar{R}_H = \frac{1}{2} \) is the solution of the Ramsey model, while \( \bar{R}_L = 1 \) is not, the higher equilibrium is more satisfactory. Another reason to select the higher equilibrium in the \( q = 1 \) case comes from the observation that, since there is no inflation in the steady state, \( R = 1 \) implies \( r = i = 0; \) i.e. an infinite money demand. In addition, it is clear from the steady-state expression of human wealth \( R = 1 \) also implies an infinite level of steady-state human wealth in the Ricardian case. It is also possible to argue that the higher steady state is the one consistent with the individual dynamics of wealth and consumption. The first thing to notice is that in a steady-state, although per-capita variables are constant, there is still some dynamics at the individual level. Since each agent is born with zero non-human wealth, in order to have a steady-state with positive per-capita non-human wealth, individuals must be accumulating non-human wealth, as long as they stay alive. Since in the steady-state individual human wealth, equal to per-capita human wealth, is constant, this implies that in the steady state individual total wealth is growing. Remembering that, with logarithmic preferences, individual consumption is proportional to total wealth, it is clear that, in the steady-state, individual consumption must be growing. From the individual first order conditions it is possible to show that, in order to have growing individual consumption, it must be \( R > \frac{1}{2}; \) If we start from the Ricardian equivalence case \( (q = 1); \) and then we marginally reduce \( q \) only the higher equilibrium satisfies the condition for growing consumption.

In Ganelli (2001a) it is showed that the analysis of the dynamics of the model leads to the same conclusion. In particular, it is possible to prove that, even for large deviations from Ricardian equivalence, reducing \( q \) raises (lowers) the higher (lower) equilibrium.

The above arguments imply that, even though this variable displays multiple equilibria, we have solved the problem of selecting the one around which we log-linearize. In what follows we therefore log-linearize around the higher equilibrium, exploiting the knowledge of some of the properties of this investigated in Ganelli (2001a). In particular, it will be useful to know that at the higher equilibrium we have \( R_0 = \frac{1}{2} \) for the particular case in which \( q = 1; \) while the value of \( R_0 \) in this equilibrium increases above \( \frac{1}{2} \) as we reduce \( q. \) As in OR (1995, 1996) and Ganelli (2000), the variables whose initial value is
zero will be normalized using the value of initial consumption given in (13).

2.4 Log-Linearization

The set of (per-capita) log-linearized equations that we will use to solve the model is given below, with hats denoting long-run (the period after the shock) log-deviations, and tildes denoting short-run (the period of the shock) ones. Log-linear variables are all in lower cases. The variable \( e \) denotes the log-deviation of the nominal exchange rate from its initial value. The nominal exchange rate is defined as the price of the foreign currency in terms of home currency. The price set by a typical foreign firm in a symmetric equilibrium is denoted by \( p(f) \):

\[
\begin{align*}
\bar{p} &= n \bar{p}(h) + (1 - n) e + p'(f) \\
\bar{p}' &= n [p(h) - e] + (1 - n) [p'(f)] \\
\bar{y} &= \mu \bar{p} - p(h) + e^w + g^w \\
\bar{y}' &= \mu [p'(f) + e^w + g^w] \\
\bar{p} &= \bar{y} + \frac{\mu}{\mu_i} (\bar{z} + p_i p(h)) \\
\bar{p}' &= \bar{y}' + \frac{\mu}{\mu_i} (\bar{z}' + p_i p'(f))
\end{align*}
\]

\[
\begin{align*}
\bar{u} &= \frac{(1 + q^-)}{(1 + A + A)} (1 + q) (1 + \bar{A}) \frac{R_0}{R_0 i} \frac{R_0}{q} h + q^- R_0 e + q^- (R_0 i - 1)e \\
\bar{a}' &= \frac{(1 + q^-)}{(1 + A + A)} (1 + q) (1 + \bar{A}) \frac{R_0}{R_0 i} \frac{R_0}{q} h^w + q^- R_0 e^w + q^- (R_0 i - 1)e \\
h &= (i - p + p(h)) + \frac{1}{\mu (1 + A)} \bar{y} + \frac{q}{R_0 i} \frac{R_0}{q} i + \frac{1}{1 + A} \bar{b}
\end{align*}
\]
\[ h^u = (i_p^u + p^u(f)) + \frac{1}{\mu(1 + A)} y^u i - \frac{q}{R_0 i} \rho_i \frac{1}{1 + A} b^u \]  

(24)

\[ m_i p = e_i \frac{p}{R_0 i} \left( \frac{p_i p}{R_0 i} \right) \]  

(25)

\[ m^u_i p^u = e^u_i \frac{p}{R_0 i} \left( \frac{p^u_i p^u}{R_0 i} \right) \]  

(26)

\[ m_i p = i \frac{b}{R_0} + b \]  

(27)

\[ m^u_i p^u = i \frac{b}{R_0} + b^u \]  

(28)

\[ a = (R_0 i \ 1)NFA_A i \ p + p(h) + y_i \ g \]  

(29)

\[ b^u = (R_0 i \ 1)NFA_A^u i \ p^u + p^u(f) + y^u_i \ g^u \]  

(30)

\[ NFA_A = \frac{2\mu i}{\mu} \frac{1}{(i p + p(h) + y) i} e_i \ g \]  

(31)

\[ NFA^u_A = \frac{2\mu i}{\mu} \frac{1}{(i p^u + p^u(f) + y^u) i} e^u_i \ g^u \]  

(32)

\[ e = p_i \ p^u \]  

(33)

Equations (15) to (33) are respectively the log-linearized versions of home and foreign price indexes, demand equations, labour-leisure trade equations, Euler equations, long-run human wealth, short and long-run money demand equations, long-run and short-run current account equations and the purchasing power parity equation. The long-run money demand functions (27) and (28) are, unlike in OR (1995, 1996) and in Ganelli (2000), functions of the long run log-deviation of the real interest rate. This is due to the fact that, with overlapping generations, the real interest rate is not tied down by the Euler equation in the simple way in which it was in OR (1995, 1996) and in Ganelli (2000). In this version of the model would still be in principle possible to derive an expression for the long-run real interest rate using equation (10). The long run value of the real interest rate,
however, being a function of human wealth, is affected by policies that, like
the one that we are going to consider, involve intertemporal redistribution
of taxation. This explains why, contrary to OR (1995, 1996) and to Ganelli
(2000), the long-run log-deviation of this variable is not zero in this model.

3 Macroeconomic Effects of Government Debt

The policy experiment that we consider is a temporary reduction in taxes,
financed by an increase in debt, with long-run taxes adjusting to pay for the
higher interests. We study this policy in a context of one period nominal
rigidity in the domestic currency price of home goods and in the foreign

We already know from Ganelli (2001b) that this policy increases short-
run consumption in a closed economy. In an open economy, however, the
question arises of which of the two countries benefits more from it. When,
for instance, the domestic country follows such a policy, the intuition and
the results of Ganelli (2001b) suggest that the intertemporal reallocation of
taxation would increase home short-run consumption. This would have an
expansionary effect on the other country, even if government spending is kept
constant, due to the absence of home bias in consumption. The home country
agents, on the other hand, will also discount the fact that some of them will
be alive next period and will have therefore to pay higher taxes, while foreign
agents will not have to worry about future higher taxes. Finding a reduced
form for relative consumption will allow us to assess formally how all these
different effects combine together to determine the final impact of the policy.
Solutions for relative output and the exchange rate will also be discussed.

Solving for macroeconomic variables in relative terms is common in this
literature\(^{15}\), and Ganelli (2000), where the solutions were derived in absolute
terms, could be considered an improvement with respect to this practice.
In this paper, however, solving the model in terms of differences is highly
desirable in order not to complicate the (already quite cumbersome) algebraic
expressions of the reduced forms. In addition, due to the fact that with
an internationally integrated bond market there is only one interest rate,
expressing the solution in relative terms allows us to eliminate the endogenous
variable that was not present in OR (1995, 1996) and in Ganelli (2000), i.e.
the long-run log-deviation of the real interest rate \( \beta \): The presence of this

\(^{15}\)See, for example, Hau (2000) and Tille (2001).
additional endogenous variable prevents us from explicitly deriving reduced forms for the absolute levels of the variables. However, the effect on the level of a variable can be decomposed, using Aoki’s formula

\[ x = x^w + (1 - n)(x - x^*) \]

into the sum of the world and of the relative effects. This means that we will also be able to discuss the level effects using the reduced forms derived in this paper and the knowledge of the closed economy described in Ganelli (2001b).

Log-linearization of the government budget constraint (excluding seigniorage) around the selected steady state gives:

\[ g + R_0 \dot{D} = \varepsilon + \dot{D} \]  

(34)

The policy that we are considering leaves government spending and short-run government debt unchanged. Therefore we have: \( \dot{g} = \dot{D} \); i.e. the increase in long-run government debt is equal to the reduction in short-run taxes\(^1\). This result, combined with log-linearization of the long-run government budget constraint gives: \( (R_0 - 1)\dot{D} = \dot{g} \). The latter equality implies that the effects of a reduction in short-run taxes can be formally captured by an increase in long-run taxes. The solution strategy will therefore be one of finding reduced forms in which long-run taxes appear as exogenous variables. The steps that we need to take in order to solve the model are similar to those already presented in Ganelli (2000).

We start by subtracting equation (22) from equation (21), which yields:

\[ i \dot{b} - i \dot{b}_q = i \dot{h} + q^* R_0 (\dot{e} - \dot{e}_q) \]  

(35)

Where \( i = \frac{(1 - q^*)}{(1 + \lambda + \lambda^*)R_0} \). Subtracting short-run foreign money demand from short-run home money demand, with money shocks set to zero, and using the purchasing power parity relation, we have:

\[ e = i (\dot{b} - \dot{b}_q) + \frac{1}{R_0} \dot{e} \]  

(36)

\( e \) is already stressed, in the initial steady state \( e_0 = 0 \) as well. A reduction in short-run taxes will therefore imply negative taxes, which can be thought of as a subsidy. The fact that taxes are negative does not alter our theoretical findings.

---

\(^{16}\) The variables that enter the budget government budget constraint, being all zero at the initial steady state, are normalized by initial consumption.

\(^{17}\) As already stressed, in the initial steady state \( e_0 = 0 \) as well. A reduction in short-run taxes will therefore imply negative taxes, which can be thought of as a subsidy. The fact that taxes are negative does not alter our theoretical findings.
\[ e = i (b \cdot d^i) \]  
(37)

Substituting (37) into (36) gives:

\[ e = i (e \cdot d^i) i \frac{1}{R_0} (b \cdot d^i) i \frac{1}{R_0} e \]  
(38)

Expressing \( h_i \cdot h^u \) as the difference of equation (23) and (24) we have:

\[ h_i \cdot h^u = i \cdot p + p^d + p(h) i \cdot p^f(f) + \frac{1}{\mu(1 + A)} (y_i \cdot y^f) i \frac{1}{(1 + A)} (b \cdot b^u) \]  
(39)

Combining equations (19), (20) and long run versions of equations (17) and (18) it is possible to show that:

\[ i \cdot p + p^d + p(h) i \cdot p^f(f) = \frac{\bar{A}}{\mu_i} \frac{\mu}{(1 + A)} (b \cdot d^i) \]  
(40)

and

\[ y_i \cdot y^f = i \frac{\bar{A}}{\mu_i} \frac{\mu}{(1 + A)} (b \cdot d^i) \]  
(41)

Substituting the last two relationships into (39):

\[ h_i \cdot h^u = \frac{\bar{A}^2}{(\mu_i \frac{1}{1 + A})(1 + A)} (b \cdot d^i) i \frac{1}{1 + A} (b \cdot b^u) \]  
(42)

We can now use equation (42) in (35), this yields:

\[ b \cdot d^i \cdot d^i = \frac{1}{i} \frac{\bar{A}^2}{i(1 + A)(1 + A)} (b \cdot d^i) i \frac{1}{i} \frac{1}{1 + A} (b \cdot b^u) \]  
(43)

Where \( i = 2 \) and \( i = 1 \). We can now use the latter equation in (38) in order to find a relationship between short-run relative consumption, the exchange rate and long run taxes:

\[ e = i (b \cdot d^i) \]  
(37)

Expressing \( h_i \cdot h^u \) as the difference of equation (23) and (24) we have:

\[ h_i \cdot h^u = i \cdot p + p^d + p(h) i \cdot p^f(f) + \frac{1}{\mu(1 + A)} (y_i \cdot y^f) i \frac{1}{(1 + A)} (b \cdot b^u) \]  
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(42)

We can now use equation (42) in (35), this yields:

\[ b \cdot d^i \cdot d^i = \frac{1}{i} \frac{\bar{A}^2}{i(1 + A)(1 + A)} (b \cdot d^i) i \frac{1}{i} \frac{1}{1 + A} (b \cdot b^u) \]  
(43)

Where \( i = 2 \) and \( i = 1 \). We can now use the latter equation in (38) in order to find a relationship between short-run relative consumption, the exchange rate and long run taxes:

\[ e = i (b \cdot d^i) \]  
(37)

Expressing \( h_i \cdot h^u \) as the difference of equation (23) and (24) we have:

\[ h_i \cdot h^u = i \cdot p + p^d + p(h) i \cdot p^f(f) + \frac{1}{\mu(1 + A)} (y_i \cdot y^f) i \frac{1}{(1 + A)} (b \cdot b^u) \]  
(39)

Combining equations (19), (20) and long run versions of equations (17) and (18) it is possible to show that:

\[ i \cdot p + p^d + p(h) i \cdot p^f(f) = \frac{\bar{A}}{\mu_i} \frac{\mu}{(1 + A)} (b \cdot d^i) \]  
(40)

and

\[ y_i \cdot y^f = i \frac{\bar{A}}{\mu_i} \frac{\mu}{(1 + A)} (b \cdot d^i) \]  
(41)

Substituting the last two relationships into (39):

\[ h_i \cdot h^u = \frac{\bar{A}^2}{(\mu_i \frac{1}{1 + A})(1 + A)} (b \cdot d^i) i \frac{1}{1 + A} (b \cdot b^u) \]  
(42)

We can now use equation (42) in (35), this yields:

\[ b \cdot d^i \cdot d^i = \frac{1}{i} \frac{\bar{A}^2}{i(1 + A)(1 + A)} (b \cdot d^i) i \frac{1}{i} \frac{1}{1 + A} (b \cdot b^u) \]  
(43)

Where \( i = 2 \) and \( i = 1 \). We can now use the latter equation in (38) in order to find a relationship between short-run relative consumption, the exchange rate and long run taxes:
\[ e = f i R_0 + \frac{1}{R_0} i \left( \frac{1}{2} q g e + \frac{1}{R_0} i \left( \frac{1}{2} + A \right) g e \right) \]  

(44)

Since long-run taxes are determined by the policy choices, in order to solve for relative consumption and the exchange rate we need to derive another relationship between the variables in (44), analogously to Ganelli (2000). This can be done making use of the current account equations. Using in equations (31) and (32) the log-linearization of the price indexes, (15) and (16), together with the assumption of price stickiness that allows us to set \( \varphi(h) = \varphi'(f) = 0 \); we get

\[ N^F A = \left( \frac{2 \mu i - 1}{\mu} \right)[i (1_i n) e + y] e i g \]  

(45)

and

\[ N^F A^* = i \left( \frac{n}{1_i n} \right) N^F A = \left( \frac{2 \mu i - 1}{\mu} \right)[ne + y i e] e i g \]  

(46)

Subtracting the latter two equations we get:

\[ N^F A = (1_i n)(\frac{2 \mu i - 1}{\mu})[(y i y) i (e i e) i (g i g) i e] \]  

(47)

We already know from Ganelli (2000) that subtracting the foreign from the home demand equation, using the purchasing power parity equation and the sticky price hypothesis, we get:

\[ y_i y^* = \mu e \]  

(48)

Substituting the latter equation into (47) we have:

\[ N^F A = (1_i n)(\frac{2 \mu i - 1}{\mu})[(\mu i 1)e i (e i e) i (g i g)] \]  

(49)

We now proceed to derive a relationship between \( N^F A \) and long-run variables, that can be used together with (49) to eliminate \( N^F A \). To do this, we start subtracting the foreign long-run current account equation, (29), from the home equivalent (30):

\(^{19}\)Note that for the moment we are not setting \( g = g^* = 0 \), although this is assumed in the policy experiment that we consider below.
\[ b_i b^s = (R_{0i} 1) \frac{1}{i n} NFA_i b + p^x + p(z) i p^n(z) + (y_i y^s) i (g_i g^n) \] (50)

Using again equations (40) and (41) we can express the latter in terms of NFA as:

\[ NFA = i 3 \frac{1}{n} \frac{1}{R_{0i}} (b_i b^s) + i 1 \frac{1}{n} \frac{1}{R_{0i}} (g_i g^n) \] (51)

where \( i 3 = \frac{\mu 1 + \mu}{1 + \mu} \). Using (43) to re-express \( b_i b^s \) we have:

\[ NFA = i 3 \frac{1}{n} \frac{1}{R_{0i}} \left[ (1 + A) \frac{1}{R_{0i}} (e_i e^s) i \frac{1}{i 2} \frac{1}{1 + A} (g_i g^n) \right] + \frac{1}{n} \frac{1}{R_{0i}} (g_i g^n) \] (52)

Putting the latter together with (49), we can eliminate NFA and get a second relationship between \( e_i (e_i e^s) \) and variables under the control of the government:

\[ e = i 4 (e_i e^s) i \frac{1}{i 2} \frac{1}{(2 \mu_1 1) (R_{0i} 1)} \frac{1}{\mu} \frac{1}{1 + A} (g_i g^n) + \frac{1}{\mu_1} (g_i g^n) + \frac{1}{R_{0i} 1} (g_i g^n) \] (53)

where \( i 4 = i 3 = \frac{R_{0i} 1}{i 2} \frac{1}{(2 \mu_1 1) (R_{0i} 1)} q + \frac{1}{i 4 1} \).

### 3.1 A Graphical Analysis

Before proceeding with an analytical derivation of the results of the model, it is useful to notice that equations (44) and (53) constitute a pair of simultaneous equations in the two unknowns \( e \) and \( e_i e^s \). From the discussion of the composite parameters made in what follows (see sections below), it can be proved that (44) is downward sloping in the \( (e_i e^s; e) \) space, while (53) is upward sloping. Furthermore, an increase in future domestic taxes shifts both curves rightward. It follows that the effect of the policy that we are considering can be represented graphically in Figure 2.
Figure 2: Effects on relative consumption and the exchange rate of a temporary tax-cut

An increase in future domestic taxation will always increase relative consumption, while the effect on the exchange rate depends on which curve shifts more to the right. The example represented, of a movement to the equilibrium $E_0$ to $E_1$, implies an increase in $e$ (a depreciation of the domestic currency), but the result could be opposite if (53) were to shift more.

3.2 Effects on Relative Consumption

We now illustrate the results analytically. It is possible to combine equation (53) with equation (44) in order to derive a reduced form for relative consumption as follows:

$$
e_i e^* = \frac{1}{(i s i i 4) i 2(1 + A)^{i i} i 3(2\mu i 1)(\mu i 1)(R_0 i 1)} (g_i g^*) + \frac{1}{(i s i i 4) \mu i 1} (g_i g^*) + \frac{1}{(i s i i 4) R_0 i 1} (g_i g^*)$$

(54)
where \( i_5 = \frac{\frac{R_0}{R_0 + 1}}{R_0} \cdot \frac{1}{2} q^* \).

As explained before, the effect of a reduction in domestic short-run taxes financed by debt is given by \( i \cdot \epsilon_i = \epsilon^* = (R_0 i, 1) \cdot \epsilon_i = \epsilon^* \). From equation (54), we can see that the latter is equal to \(^{20}\):

\[
(R_0 i, 1) \cdot \frac{1}{i \cdot \frac{1}{2} (1 + A)} \cdot \frac{1}{i \cdot \frac{1}{2} (1 + A)} \cdot \frac{1}{i \cdot \frac{1}{2} (1 + A)} \cdot \frac{1}{i \cdot \frac{1}{2} (1 + A)} \cdot \frac{1}{i \cdot \frac{1}{2} (1 + A)} (55)
\]

In order to evaluate the final effect of the policy that we are considering we need therefore to determine the sign of the expression given in (55). This task does not look straightforward, but our knowledge of the initial steady state can help in it. We know that in the initial steady-state that we selected \( R_0 > \frac{1}{2} > 1 \); this ensures that \( (R_0 i, 1) = r_0 > 0 \). Furthermore, it implies that \( i_3 > 0 \); since given the parameters we know that \( i_3 > 0 \); we can also conclude that the magnitude in the square bracket in equation (55) is negative. Assuming for the moment that \( i_2 > 0 \); we have \( \frac{1}{i \cdot \frac{1}{2} (1 + A)} < 0 \): If \( i_2 > 0 \) is true, therefore, the expression in (55) is the product of two negative terms (the one in the square bracket and \( (R_0 i, 1) \cdot \frac{1}{i \cdot \frac{1}{2} (1 + A)} \)) and of a positive one \( (\frac{1}{i \cdot \frac{1}{2} (1 + A)} \): In this case the policy that we are considering would have a positive effect on relative consumption. In order to establish this result we need therefore to prove that \( i_2 \) is positive. To demonstrate this, let’s remember that in the cases in which the agents have infinite lives \( R_0 \) takes the value \( \frac{1}{2} \) in the initial steady state around which we are log-linearizing. Although we can not set \( q \) exactly equal to 1 without making the model collapse, substituting \( R_0 = \frac{1}{2} \) in \( i_2 \) can give some insight on what the effect will be in situations in which the deviation from Ricardian equivalence is not too large. Noticing that \( i_2 \) can be written as:

\[
i_2(R_0) = 1i \cdot \frac{(\frac{1}{i} \cdot \frac{q^*}{q})}{(1 + A + A)} \cdot \frac{R_0}{R_0 i} \cdot \frac{\tilde{A}^2}{q(i) \cdot \frac{1}{(1 + A)}}
\]

we can evaluate its value at \( R_0 = \frac{1}{2} \) as:

\[
i_2(\frac{1}{2}) = 1i \cdot \frac{(\frac{1}{i} \cdot \frac{q}{q})}{(1 + A + A)} \cdot \frac{\tilde{A}^2}{(\frac{1}{(1 + A)}) \cdot \frac{1}{(1 + A)}}
\]

\(^{20}\)We are now keeping government spending fixed in both countries in every period. This implies \( g = g^* = b = b^* = 0 \):

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and the latter expression is clearly positive: A generalisation of this result follows if we notice that, holding other things constant, \( i_2(R_0) \) is increasing in \( R_0 \) (because \( \frac{R_0}{R_0} \) is decreasing in \( R_0 \)): Therefore, when we remove the "approximation" that \( R_0 = \frac{1}{2} \); by restoring \( R_0 \) to its true value that is bigger than \( \frac{1}{2} \); we simply strengthen the conclusion that \( i_2(R_0) > 0 \): We can therefore conclude that the home country would benefit more than the foreign country, in terms of increased consumption, from implementing the policy that we are considering.

### 3.3 Effects on Relative Output and the Exchange Rate

We already know, from (48), that there is a simple relationship between relative output and the exchange rate. Solving for one of these two variables, therefore, also provides a reduced form for the other. The easiest way to do it is to put together equations (44) and (53), eliminating relative consumption from them. This yields the following reduced form for the exchange rate:

\[
e = \frac{1}{i_6 i_7 (1 + A)} \left( \frac{i_6}{i_2 R_0} + \frac{i_3}{i_2} \gamma \frac{\mu}{(\mu_i - 1)(R_0 - 1)} \right) (g_i g^\mu) + \frac{1}{i_6 i_7} \left( g_i g^\mu \right)
\]

Where \( i_6 = i_7 = \frac{1}{4} \): The last equation and equation (48) imply that the effects of the policy that we are considering on the exchange rate and on relative consumption are given by:

\[
i \otimes \otimes = (R_0 i) \otimes \otimes = \frac{1}{i_6 i_7 (1 + A)} \left( \frac{i_3}{i_4} \frac{\mu}{(\mu_i - 1)(R_0 - 1)} + \right)
\]

and
To determine the effect of a temporary reduction in taxes, formally captured by an increase in long-run taxes, let's notice that we already know that \( \mu \frac{R_0 - 1}{R_0 - 1} \frac{1}{\mu} \frac{1}{\mu + 1} \frac{1}{\mu + 1} \) < 0 in the selected steady state. The sign of the effect will therefore depend on the sign of the expression in square brackets in equations (57) and (58). Let's notice that, with simple algebraic rearrangements, the latter can be shown to be equal to:

\[
\frac{1}{R_0^{1/2} q} + \frac{(R_0 - 1)(2\mu - 1)}{\mu} i \frac{1}{R_0^{1/2} q} + (R_0 - 1) \] (59)

(59) will be bigger than zero as:

\[
\frac{2\mu - 1}{\mu} \frac{1}{\mu} \frac{1 + \bar{A} \mu}{1 + \bar{A}} = i_3
\]

It is interesting to notice that the left-hand side and the right-hand side of the above inequality are equal when \( \bar{A} = 1 \); In that case, therefore, the effect on the exchange rate and on relative output is zero. Furthermore, \( i_3 \) can be re-written as \( \mu \frac{(\mu - 1)^2}{\mu + 1 + \bar{A}} \); which is increasing in \( \bar{A} \); it follows that (59) is bigger than zero when \( \bar{A} > 1 \); and viceversa. This implies that the effect on the exchange rate of the country that expands is positive (i.e. a depreciation) when \( \bar{A} < 1 \); and negative (an appreciation), when \( \bar{A} > 1 \). Accordingly, relative output will increase as \( \bar{A} < 1 \); and decrease as \( \bar{A} > 1 \). It is useful to stress how the latter condition is basically comparing the weight on leisure in the utility function (\( \bar{A} \)) with the weight on consumption (equal to 1). As already emphasized at the beginning of the paper, the dependance of these effects on the parameters of the utility function conınrs how the specification of the microfoundations can be crucial in determining the results in this class of models. This in turn reinforces our belief that the exercises that we are conducting using the OR framework can be a useful contribution in the debate among researchers, in order to help achieving some convergence on a preferred specification of the model.

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In trying to contrast our results to the pre-existing literature, a natural point of reference is the two-country version of the MF model. Although, given the static nature of the latter, it is not possible to differentiate debt-financed from balanced-budget expansions, the effect of a fiscal shock in MF is still an interesting benchmark for comparison. In MF a fiscal expansion brings about an appreciation of the exchange rate. This result emerges in our model in the case in which the weight on leisure is bigger than the one on consumption in the utility function ($\alpha > 1$):

In numerical examples used in previous papers (Ganelli 2001a; Ganelli 2001b) we have regarded $\alpha < 1$ as the most plausible case. This formalizes the assumption that leisure brings less utility than consumption. A very low $\alpha$ is also the case more continuous with the neo-classical idea of the existence of a "natural" level of output. Assigning numerical values to the parameters of a utility function is, however, a difficult exercise, and we have no reason to rule out completely the case in which $\alpha > 1$; at least theoretically. Our results, therefore, seem to suggest that a more "Keynesian" (in the sense of the consistency with the ad hoc MF framework) result emerges as agents value leisure more than consumption in the utility function.

It is also useful to notice that in this model the correlation between relative consumption and the exchange rate, brought about by a fiscal shock, can be opposite to the one observed in Ganelli (2000). While in Ganelli (2000) a balanced-budget expansion, decreasing relative consumption and therefore reducing demand for the home currency, would unambiguously raise $e$ (depreciate the exchange rate), here an increase in $e$ can be consistent with an increase in relative consumption (when $\alpha < 1$): The emergence of a positive correlation between relative consumption and the exchange rate suggests that, once we depart from Ricardian equivalence, mechanisms other than the dependence of money demand on consumption can be at work in determining the effect on the exchange rate. This is a stark difference with the working of the model highlighted in Ganelli (2000). The fact that the result can be so different, depending on whether the fiscal shock is financed by lump-sum taxes or by debt, confirms the value of building a version of the model in which Ricardian equivalence does not hold.

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21 Note that when $\alpha = 0$ the agents inelastically supply all their endowment of time.
3.4 Effects on the Levels of the Variables

As already stressed above, in a two-country model like the one we are presenting the level of a generic domestic variable can be decomposed as follows:

\[ x = x^w + (1 - n)(x - x^\text{f}) \]  
(60)

while the level of a foreign variable is given by:

\[ x^\text{f} = x^w - n(x - x^\text{f}) \]  
(61)

It follows that some considerations about the effects on the levels of variables can be done, even without explicitly solving them in reduced forms, on the basis of the knowledge of the reduced forms for relative variables, and of the results derived in Ganelli (2001b) for a closed economy. The world economy is here the sum of the home and of the foreign country. Perturbations that affect only one country's policy variables, leaving the other country's ones unchanged, will therefore affect world output and consumption in the same way as in Ganelli (2001b).

The latter ensures that a temporary reduction in taxes has a positive effect on world consumption \( \Delta W \) in this model. Combining the latter with our result of a positive effect on relative consumption, equation (60) implies a positive internal multiplier for the level of consumption. What about the international spillover on consumption of such a policy? Formula (61) shows how the final effect on \( \Delta x \) of a domestic fiscal expansion is the sum of the positive world effect minus a proportion of the positive effect on the differential (home minus foreign) consumption. While this does not allow us to establish in a rigorous way the sign of the spillover, it shows how the relative size of the two countries matters. The positive world effect is in fact more likely to dominate as \( n \) decreases. An intuition for this can be found in the fact that, as the dimension of the domestic country becomes smaller, the proportion of private domestic consumption that falls on foreign goods increases, and this should have a positive effect on foreigners' wealth and, therefore, on their consumption. A positive effect on foreigners' consumption is also consistent with the intuition that the (future) tax bill deriving from the current tax cut, being paid only by the domestic residents that will be alive in the next period, will not be a burden for foreign residents.

The effects on the levels of output depend, of course, on whether \( \bar{A} \) is bigger or smaller than unity. We know from Ganelli (2001b) that a debt
nanced tax-cut increases world output as well as consumption when government spending is constant. In the $\bar{A} < 1$ case therefore a domestic depreciation brings about an increase in relative output that, combined with the positive effect on world output, gives a positive multiplier for domestic output. If $\bar{A} > 1$ the effect on relative output is negative, and the two terms in equation (60) go in the opposite direction. This means that the final multiplier on domestic output is ambiguous.

The effects of $\bar{A}$ on the international output spillover are somewhat reversed. In the $\bar{A} < 1$ case, infact, with a positive output differential, equation (61) implies an ambiguous spillover. When $\bar{A} > 1$, conversely, the output spillover is unambiguously positive.

The effects on the levels of the variables discussed above are summarized in Table 1.

### Table 1. Effects of a temporary domestic tax cut on the levels of short-run variables

<table>
<thead>
<tr>
<th></th>
<th>$\bar{A} &lt; 1$</th>
<th>$\bar{A} &gt; 1$</th>
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<tbody>
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<td>$e$</td>
<td>$+$</td>
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<tr>
<td>$e^c$</td>
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<td>ambiguous</td>
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<td>$y$</td>
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<td>$y^c$</td>
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</table>

**4 Conclusions**

This paper combines the OR (1995,1996) framework with an overlapping generations structure of the Blanchard (1985) type. To the best of the author’s knowledge, this paper is the first contribution in the literature to analyse the effects of government debt in the New Open Economy Macroeconomics framework. Our analysis highlights how the real effects of debt in a two country model can depend on the microeconomic preference structure. Analytical results suggest that a temporary reduction in taxation by the domestic country, matched by an increase in long-run taxes to meet the increased interest burden, unambiguously raises relative consumption. The effect on the nominal exchange rate, and therefore on output, depends on the relative weights of consumption and output in private utility. The paper also discusses the effects on the absolute levels of output and consumption.
References


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