Skill Premium Effects on Fertility and Female Labor Supply

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Abstract: In the last twenty years the U.S.A. has seen a recovery of total fertility rates and a continuous increase in the female labor supply. This increase has been particularly spectacular among women with newborn children. We construct a general equilibrium overlapping generations model able to capture this changing relationship (from negative to positive) between fertility and female labor supply. The factor behind this result is the increase of the skill premium and the relative decline of the market price of childbearing, which induces skilled women to substitute market activities for non-market activities (childcare) without giving up having children if the income effect dominates the substitution effect.

JEL classification: J10, J22

Key words: Demographic transition, female labor supply, fertility rates, child care, skill premium.

1 Introduction

Since the Second World War, fertility and female labor supply have been negatively related in all developed countries (see, e.g., Ahn and Mira (1999), Hotz et al. (1997), Willis (1987)). This negative relationship has been
explained in literature by using the theory of allocation of time developed by Mincer (1963) and Becker (1965). Mincer (1963) emphasizes the relevance of the opportunity cost in evaluating the cost of children and relates it to women’s time, consequently, to women’s wage rate. Becker (1965) develops a formal theory of the allocation of time. Willis (1973,1987) mentions these two works as the origin of the “female cost of time hypothesis” to give an explanation for the negative relationship between fertility and income (see also the survey in Hotz and Miller (1997)). According to that hypothesis, it is assumed that childbearing is a time-intense activity, provided by females, and when “women do participate in the market, the cost of time is determined by their (marginal) wage rate”\(^1\). Therefore, the higher the market wage rate the higher the opportunity cost of childbearing, which induces an increase in female labor supply and may induce a decrease in fertility demand if the substitution effect dominates the income effect.

However, as pointed out by Ahn and Mira (1999), since the second half of eighties in some OECD countries (Sweden, Norway, Denmark, Finland and USA), a recovery of total fertility rates has occurred with no decrease in female labor supply. In fact, the female labor supply has maintained its upward trend, with most of its increase being due to the increase of labor supply from women with newborn children. In particular, in the U.S.A., the Total Fertility Rates (TFR) decreased during the period 1960-1980, increased in the second half of the eighties and established in the nineties, at the replacement level. In addition, Hotz et al. (1997) remark that during the period 1975-1994, the labor force participation rate increased by 36% for married women, and by 91% for married women with children aged one or younger. Hotz et al. (1997) mention “the increased availability of market substitutes (i.e. child care) for the mother’s time” as a possible explanation for this phenomenon.

On the other hand, since the eighties, there has been a substantial increase in the skill premium in the U.S.A.. In the last twenty years the skill premium has increased, on average, by 20%.. At the same time, over these last twenty years, the price per unit of child care services has decreased relatively with respect to the average women’s wage, and there has been an increase in the use of formal child care services by employed mothers in the U.S.A..\(^2\): For example, we can see that market child care arrangements used for preschoolers by families with employed mothers increased substantially between 1977 and 1994. Furthermore, the increase in the use of market child care is much higher for skilled women than for unskilled women (see, for
We build a general equilibrium overlapping generations model in which, as in Galor and Weil (1996), we incorporate the “female cost of time hypothesis” into a growth model, endogenizing, in particular, the increase in the skill premium. We will consider, as in Krusell et al. (1997), that the increase in the skill premium is due to a higher complementarity between physical capital and skilled labor than with respect to unskilled labor. Consequently, as the economy grows, the skill premium will be higher. On the other hand, as the skill premium increases, so does the opportunity cost of childbearing time, particularly for skilled women. Therefore, they will substitute market activities for non-market activities (childbearing), i.e. skilled women will increase their labor supply at the expense of less home production time. If the price of child-care is too expensive in terms of the wage rate they receive, then they will decide to have less children (the substitution effect dominates). However, as the price of child care declines relatively to household income, women may increase their labor supply and have more children by purchasing childbearing time in the market (the income effect dominates). We are going to assume that this relative decline of the market price of child care with respect to women’s mean wage will be caused by the increase of the skill premium and by the fact that the child care sector is driven mainly by unskilled labor.3 Since we are considering children as a normal good, the higher the skill premium, the higher the income effect with respect to the substitution effect because household income increases more than the market price of child care with the skill premium. The substitution effect will be lower the cheaper the market child care is.

This mechanism has already been proposed by Ahn & Mira (1999). They point out that the increase in female educational attainment, and as a consequence, the increase in higher-wage occupations could have lessened the price of market child care for these women, with this effect being higher due to the fact that the childcare sector is driven mostly by unskilled labor. In consequence, the substitution effect relative to the income effect of the demand for children would decrease when the wage rate increases.

There are some studies that have analyzed the relationship, in the last twenty years, between the cost of market child care and the fertility decision at micro level. For example, Ermisch (1998) develops a theoretical and empirical analysis of family size decisions and the cost of market child care. His “theoretical model implies that the impact of the mother’s wage on her completed fertility varies with the market price of childcare, and that this effect
increases (becoming less negative or more positive) with the level of her wage. Econometric analysis of British micro-data confirms the main predictions (page 79). Blau and Robins (1988, 1989) find a negative effect of the relative increase of market childcare on fertility and non-market activities that women supply, using U.S. data. Blau and Robins (1998) show the relative decrease of the market childcare price in the last twenty years in the U.S.A.. Leibowitz and Klerman (1995) show the high participation rate of married women with preschool children.

At the macro level, Conesa (1999) analyzes how the transition from an early childbearing regime to a late childbearing regime may explain the evolution of the total fertility rates in the last thirty years in the U.S.A.. This transition seems to be determined by the increase in the educational attainments of women given the differences in the timing of fertility decisions by households, conditional on their educational attainment. However, Conesa (1999) does not study the increase in female labor supply in these last twenty years in the U.S.A.. Regalia and Rios-Rull (1999) quantify the effect of the wage gap within and between sexes in the last thirty years on the increase in single mothers and the stability of fertility in the nineties in the U.S.A.. Olivetti (2000) measures the contribution of the increase in returns on labor experience to the increase of the married female’s life cycle profile of hours, and in particular on the hours worked by women with preschool children. However, Olivetti (2000) does not endogenize the fertility decision. Caucutt et al. (2001) explain the increase in the female labor supply and the recovery of the TFR as a result of the increase in the return on women’s labor experience. They endogenize women’s fertility and marriage decisions and find that women born in the sixties postponed their fertility time due to the higher return on their labor-market experience.

The paper is organized as follows. In the second section we present some empirical evidence, in the third section we provide a very simple overlapping generations model able to capture the changing relationship between fertility and female labor supply, from a negative to a positive relationship, as the economy grows. In the last section, we present our conclusions.

2 Some Empirical Evidence

This section reports some empirical evidence regarding women’s labor force participation and fertility rates in the U.S.A. in the last thirty years. Also,
we show some evidence for some factors that may be behind the changing relationship (from negative to positive) between fertility and female labor supply.

In table I we show the decrease in Total Fertility Rates in the sixties and seventies, the recovery in the eighties and the stability in the nineties.

**TABLE I.- TOTAL FERTILITY RATES IN THE U.S.A.**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>3.0</td>
<td>2.8</td>
<td>2.5</td>
<td>1.7</td>
<td>1.8</td>
<td>1.8</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>


In table II, we show the continuous increase in women’s participation rates since the sixties.

**TABLE II.- WOMEN’S PARTICIPATION RATES IN THE U.S.A.**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>37.8</td>
<td>43.4</td>
<td>51.1</td>
<td>58.9</td>
<td>61.8</td>
<td>69.0</td>
<td>70.5</td>
</tr>
</tbody>
</table>


In Table III we show that the highest increase in the female participation rate has been for women with a child of one (or younger). In the last column we can see that the labor supply for married women with children under one year old increased by 91% while the increase in the labor supply for all wives is only 36%.

**TABLE III.- Labor force participation rates for wives, husband present, by age of own youngest child in the US: 1975-1994.**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All wives</td>
<td>44.4</td>
<td>50.2</td>
<td>54.2</td>
<td>58.2</td>
<td>60.6</td>
<td>36%</td>
</tr>
<tr>
<td>1 year or under</td>
<td>30.8</td>
<td>39.0</td>
<td>49.4</td>
<td>53.9</td>
<td>58.8</td>
<td>91%</td>
</tr>
<tr>
<td>2 years</td>
<td>37.1</td>
<td>48.1</td>
<td>54.0</td>
<td>60.9</td>
<td>64.5</td>
<td>74%</td>
</tr>
<tr>
<td>3-5 years</td>
<td>42.2</td>
<td>51.7</td>
<td>58.4</td>
<td>64.1</td>
<td>64.6</td>
<td>53%</td>
</tr>
<tr>
<td>6-13 years</td>
<td>51.8</td>
<td>62.6</td>
<td>68.2</td>
<td>73.0</td>
<td>75.5</td>
<td>46%</td>
</tr>
<tr>
<td>14-17 years</td>
<td>53.5</td>
<td>60.5</td>
<td>67.0</td>
<td>75.1</td>
<td>77.2</td>
<td>44%</td>
</tr>
<tr>
<td>No children under 18</td>
<td>43.8</td>
<td>46.0</td>
<td>48.2</td>
<td>51.1</td>
<td>53.2</td>
<td>21.5%</td>
</tr>
</tbody>
</table>
In Table IV we can see that the increase in working women with children under 2 years old is higher for skilled women.

Table IV: Working married women with children under 2 years old.

<table>
<thead>
<tr>
<th></th>
<th>1967</th>
<th>1992</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>% skilled working women w/ children under 2</td>
<td>33%</td>
<td>60%</td>
<td>81%</td>
</tr>
<tr>
<td>% unskilled working women w/children under 2</td>
<td>37%</td>
<td>50%</td>
<td>35%</td>
</tr>
</tbody>
</table>

Source: PSID. Women are between 18 and 40 years old.

With respect to the use of child care, we show, in Table V, that the use of "organized child care" has increased in the last twenty years. In the last column we can see the change, in percentage terms, in the period 1977-94 in the different types of child-care arrangements. The most relevant fact is that the fraction of families with employed mothers that use organized child care facilities for their preschoolers has grown in the last twenty years more than would be proportional to the increase in other uses of child-care. The increase in organized childcare facilities is 430% while the increase in the number of children that use any type of child-care is only 135%.

TABLE V.- Type of arrangement of Child Care.

<table>
<thead>
<tr>
<th>Type arrang.</th>
<th>1977</th>
<th>1994</th>
<th>Change in per cent terms</th>
</tr>
</thead>
<tbody>
<tr>
<td># children</td>
<td>4370</td>
<td>10288</td>
<td>135%</td>
</tr>
<tr>
<td>Care in child’s home</td>
<td>1481</td>
<td>3395</td>
<td>129%</td>
</tr>
<tr>
<td>Care in another home</td>
<td>1778</td>
<td>3220</td>
<td>80%</td>
</tr>
<tr>
<td>Organized child care</td>
<td>568</td>
<td>3025</td>
<td>430%</td>
</tr>
<tr>
<td>Child care at work</td>
<td>498</td>
<td>566</td>
<td>13%</td>
</tr>
</tbody>
</table>

Source: U.S. Bureau of the Census

3 The Model

In this section we propose a general equilibrium model based on Galor and Weil (1996). The model proposed uses the "female cost of time hypothesis" developed by Mincer (1963) and Becker (1965) where variations in women’s wage affect female labor supply and childbearing decisions. Children are considered as a normal good.
The model proposed is an overlapping generations model whose household lifetime horizon is finite (two periods) and not altruistic. The production sector consists of two sectors: the final good sector and the child care services sector. In this model we endogenously obtain the increase of the availability and the use of formal child care services as the economy grows and we study how their availability may determine the household’s fertility and female labor supply choices. The mechanism driving our results is the fall of the market price for child care in relation to average female wages. The assumption that child care services are mainly driven by an unskilled labor force implies that an increase in the skill premium will bring down the market price for childbearing relative to the wage rate of skilled labor. This model is close to that of Galor and Weil (1996) except for the introduction of the child care sector.

The model we construct assumes perfect certainty and perfect capital markets, which implies that in individuals with rational expectations the completed family size will depend exclusively on the household’s lifetime earnings and prices. The time path of household income will play no role in the optimal timing of childbearing. Consequently, we do not need to make any distinction between Total Fertility Rate and Completed fertility.

3.1 The Problem of the Household

We consider that each household comprises two adults (man and woman) and the couples of children they have. As in Galor and Weil (1996) we assume that households derive utility only from the number of children they have and from consumption in their last period of life. Children are a normal good.

Alike Galor and Weil (1996), we take the division of labor exogenously. Adult households live for two periods and, during their first period, the women of those households take fertility and labor decisions. In their second period, households live on their savings. Only females can decide how to allocate their (skilled) time between market and household activities. For the sake of simplicity, we consider that females are endowed with $\mu_F$ units of unskilled time (which are supplied inelastically to the labor market) and $(1 - \mu_F)$ units of skilled time. Analogously, men are endowed with $\mu_M$ units of unskilled time and $(1 - \mu_M)$ units of skilled time that they supply inelastically to the labor market. They are not allowed to provide household activities.
Each adult household faces the following problem\textsuperscript{12},

\[
\max_{f x_t, n_t, s_t, c_{t+1}} u_t = \max_{f x_t, n_t, s_t, c_{t+1}} f(n_t) + (1 \mu) \ln(c_{t+1})
\]

s.t.:

\[
s_t + [p_{ch} x_t + w_t^s (1 \mu)] s_t = w_t^s (2 \mu) + w_t^u \mu
c_{t+1} = (1 + r_{t+1}) s_t
\]

\[
x_t \in [0; 1]
\]

\[
(1 \mu_F) , (1 \mu_F) z_t , 0
\]

where,

- \(c_{t+1}\) denotes household consumption in the second period of adult life.
- \(s_t\) represents the household savings in the first period of adult life.
- \(n_t\) denotes the number of children the household decides to have along its lifetime horizon.
- \(z\) denotes the required fraction of childbearing time in the first period of adult life per couple of children born.
- \(x_t\) denotes the fraction of \(z\)'s time bought on the child care market. Home and market childcare time are perfect substitutes\textsuperscript{13}.
- \((1 \mu_F)\) is the total endowment of women's time available for childbearing activities.
- \(\mu = (\mu_F + \mu_M)\) denotes women's and men's endowment of unskilled time.
- \(w_t^u\) denotes the wage rate per unit of unskilled labor.
- \(w_t^s\) denotes the wage rate per unit of skilled labor.
- \(p_{ch}\) denotes the market price of child care per unit of time and couples of children.

Solving the household's problem, we have that the use of formal child care and the number of children that maximize household's utility are given by\textsuperscript{14},

\[
(x_t; n_t) = \begin{cases} 
(0; 1) & \text{if } w_t^s < p_{ch} \\
(2; 0) & \text{if } w_t^s = p_{ch} \\
(x_t; n_t) = 0 & \text{if } w_t^s > p_{ch} \\
\end{cases}
\]

(1)
Expression (1) shows the number of children and childbearing time bought on the child care market that maximize the household’s utility, depending on the wage rate of skilled time relative to the market price of childbearing per unit of time and couples of children. The number of children demanded by households are a fixed proportion of the ratio of total (potential) income to the price of child care.

Market childbearing and mother’s childbearing are perfect substitutes, however, the price of childbearing will depend on the type of childbearing (mother’s or market). If no market child care is used, then the price of childbearing time is the opportunity cost of allocating the mother’s skilled time to childbearing activities instead of working (i.e. the wage per unit of skilled labor). In this case, as the wage per unit of skilled labor increases (or the skill premium, i.e. \( w^s_t = w^m_t \)), the demand of children decreases and female labor supply increases (the substitution effect dominates the income effect in fertility demand).

Market child care is used if the market price of childbearing per unit of time is no greater than the price per unit of (skilled) time allocated to the market. If the market price of childbearing equals the wage rate of skilled labor, women will be indistinguishable about the allocation of their skilled time between household (childbearing) and market activities (\( x_t \in [0; 1] \)).

As the wage per unit of skilled labor increases relative to the market price of childbearing, the demand of couples for children increases (the income effect dominates the substitution effect) and the female labor supply remains constant at its highest level \( = 1 \). See Appendix 1 for a graphical solution.

Therefore, female labor supply (unskilled (supplied inelastically) and skilled labor) is given by

\[
fls_t = \begin{cases} 
8 \frac{(1 - zn_t)}{wn_t} & \text{if } w^s_t < p_{ch_t} \\
9 \frac{1 - zn_t(1 - x_t)}{wn_t} & \text{if } w^s_t = p_{ch_t} \\
1 & \text{if } w^s_t > p_{ch_t}
\end{cases}
\]

where \( l^s_t \) is always non-decreasing in \( w^s_t \).

Taking into account households’ optimal choices in the demand for children, market child care and female labor supply, household savings are given by,
\[
\begin{align*}
8 & \quad w_t^s(1_i \mu_M) + w_t^u|\mu; \quad \text{if } w_t^s < p_{ch_t}; \quad x_t = 0 \text{ and } z_{nt} = (1_i \mu_F) \\
9 & \quad (1_i \mu)[w_t^s(2_i \mu) + w_t^u|\mu]; \quad \text{if } w_t^s < p_{ch_t}; \quad x_t = 0 \text{ and } z_{nt} < (1_i \mu_F) \\
\text{s}_{nt} = & \quad (1_i \mu)[w_t^s(2_i \mu) + w_t^u|\mu]; \quad \text{if } w_t^s = p_{ch_t}; \quad x_t = 2 (0; 1) \\
& \quad (1_i \mu)[w_t^s(2_i \mu) + w_t^u|\mu]; \quad \text{if } w_t^s > p_{ch_t}; \quad x_t = 1 \\
\end{align*}
\]

where we observe that \text{s}_{nt} is always increasing in \(w_t^s\):

Figures 1 and 2 in Appendix 1 show these households’ optimal choices. It is straightforward to see that the demand for children shows a different behavior depending on the relative market child care with respect to the skilled wage. Firstly, if market child care is not used we obtain a Beckerian behavior characterized by a negative relationship between increases in household income (or female labor supply) and demand for children. However, as market child care is used we obtain a positive relationship between household income, female labor supply and demand for children.

### 3.2 Production Sector

There are two production sectors in the economy: the final good sector and the child-care sector.

#### 3.2.1 Child-Care Sector

We will assume that the only factor employed in the child care sector is unskilled labor\(^{15}\). If we write the production function per young households in period \(t\):

\[
ch_t = \lambda l_{ch_t}^u
\]

where,

- \(ch_t\) represents the output of child care services provided per young household in period \(t\).
- \(l_{ch_t}^u\) represents the total amount of unskilled labor employed in the child care sector per young household in period \(t\).
½is a parameter that affects total productivity in the child-care sector.

Under perfect competition we have,

$$p_{ch} = (w_t^u)^{1/2}$$

3.2.2 The Final Good Sector

The technology of the final good sector in the economy is given by the following aggregate production function,

$$Y_t = (1 - b) [\alpha K^{eta} + (1 - \beta)(hL^S)^{\gamma}](1 - \gamma) + b L^U_{y,t}$$

The first term of this production function is a C.E.S. function with an elasticity of substitution between physical capital and skilled labor of $1 - (1 - \gamma)$. Since we want to consider that an increase in physical capital raises the skill premium, we have to assume that skilled labor is more complementary to the stock of physical capital than unskilled labor, i.e. $\gamma < 1$. Rewriting the production function in terms of young households in period t we have

$$y_t = (1 - b) [\alpha K^{eta} + (1 - \beta)(hL^S)^{\gamma}](1 - \gamma) + b y^u_{t}$$

and the corresponding prices given that the sector behaves competitively,

$$r_t = (1 - b) [\alpha K^{eta} + (1 - \beta)(hL^S)^{\gamma}](1 - \gamma) \cdot k$$

$$w^s_t = (1 - b)(1 - \beta)[\alpha K^{eta} + (1 - \beta)(hL^S)^{\gamma}](1 - \gamma) \cdot h$$

$$w^u_t = b$$

The skill premium is defined as the ratio of the wage per unit of skilled labor to the wage per unit of unskilled labor, therefore the skill premium can be expressed as:

$$\frac{1}{\gamma} \cdot \frac{w^s_t}{w^u_t} = \frac{w^s_t}{b}$$
It is straightforward to show that,
\[
\frac{\partial q}{\partial t} > 0; \quad 8k_t > 0, \quad \text{and} \quad \frac{\partial q}{\partial t} < 0; \quad 8l_t^s > 0
\] (6)

where \( l_t^s = (2 \mu_i \ i z_i(n_t(1 \ i x_t)). \) And,
\[
\frac{\partial s}{\partial t} < 0; \quad 8n_t > 0, \quad \text{and} \quad \frac{\partial s}{\partial x_t} > 0; \quad 8x_t > 0
\] (7)

3.3 Competitive Equilibrium

A competitive equilibrium is a sequence of allocations \( \{c_{t+1}; x_t; n_t; s_t; Y_t; K_t; L_t^S; L_t^U; Ch_t; L_t^{UL}; \} \) and prices \( \{r_t; w_t^S; w_t^U; p_{ch_t} \} \) such that,

1) \( \{c_{t+1}; x_t; n_t; s_t \} \) solve the household’s maximization problem by taking prices as given,

2) \( \{Y_t; K_t; L_t^S; L_t^U \} \) solve the good sector’s ..rns maximization problem by taking prices as given,

3) \( \{Ch_t; L_t^{UL} ; \} \) solve the child care ..rns’ maximization problem by taking prices as given,

4) All markets clear in every period,

i) Good market,
\[
Y_t = C_t + (K_{t+1} \ i (1 \ i \ \pm)K_t)
\]
\[
(r_t + \pm)K_t + w_t^SL_t^S + w_t^UL_t^U =
\]
\[
(1 + r_t)s_t; L_t^S + (w_t^S L_t^S + w_t^UL_t^U) \ i \ p_{ch_t} z_t x_t L_t
\]
\[
i s_t L_t + (K_{t+1} \ i (1 \ i \ \pm)K_t)
\]
assuming that the child care market is in equilibrium (iv),

\[ p_{cht}z_t x_t L_t = p_{ch} Ch_t = w_t^U L_t^U \]

\[ = w_t^U (L_t^U \cdot L_t^U) \]

and that the economy sets out with a stock of physical

\[ K_0 = s_1 L_0 \]

we have that,

\[ K_{t+1} = s_t L_t \]

where \( L_t \) is the number of young adult households in period \( t \).

ii) Unskilled labor market,

\[ \mu L_t = L_t^U + L_t^U = L_t^U \]

Therefore, as long as market child care is not demanded, unskilled labor will be working in the good sector, and no child care market will be provided. Child care services will be provided only when the skill premium is high enough.

iii) Skilled labor market:

\[ [2 \mu z(1 x_t) n_t] L_t = L_t^S \]

iv) Child Care market,

\[ Ch_t = z x_t n_t L_t \]

Finally, we have that \( GDP_t = Y_t + p_{cht} Ch_t \):

\[ G D P_t = Y_t + p_{cht} Ch_t \]

## 3.4 Dynamics

The stock of physical capital at period \( t+1 \) will be given by the savings of the young population at the end of period \( t \),

\[ K_{t+1} = s_t L_t \]
rewriting that condition in terms of the number of young households in period \( t \),

\[
k_{t+1}n_t = s_t
\]

Taking into account households’ optimal choices of the demand for children, market use of child care and savings we can obtain the physical capital per young household at time \( t + 1 \) as

\[
k_{t+1} = \frac{s_t}{n_t} = \text{(9)}
\]

We can substitute \( w^s_t \) and \( w^u_t \), which depend on \( k_t, x_t(k_t) \) and \( l_t^s(n_t) \), where the last two also depend on \( w^s_t \) and \( w^u_t \), we can obtain an equilibrium dynamic for the physical capital per young household taking into account the equilibrium choices of all economic agents in the economy (see Appendix 4),

\[
k_{t+1} = \frac{s_t}{n_t} = \hat{A}(k_t) = \text{(10)}
\]
8 \[
\frac{\hat{z}}{\hat{41} \hat{b}}(1 \hat{i} \ b)(1 \hat{i} \ b)h^2(1 \hat{i} \ b)(1 \hat{i} \ b)(h(1 \hat{i} \ b))^{-\frac{1}{2}}(1 \hat{i} \ b) + b \mu \hat{g}
\]

9 \[
\frac{\hat{z}(1 \hat{i} \ b)}{\hat{46}}h^2(1 \hat{i} \ b)(1 \hat{i} \ b)h^2(1 \hat{i} \ b)(1 \hat{i} \ b)(h(1 \hat{i} \ b))^{-\frac{1}{2}}(1 \hat{i} \ b) + b \mu \hat{g}
\]

Therefore, given an initial value for \(k_0\), the dynamic evolution of the equilibrium \(k_t\) is determined by \(\hat{A}(k_t)^{17}\).

Also, we can obtain the equilibrium dynamic of the endogenous completed fertility and female labor supply decisions. In this way, we will have,

\[
\hat{z}_{n_t} = \hat{1}(k_t) \quad \text{if} \quad k_0, k_t \geq 0
\]

\[
\hat{1}(k_t) \quad \text{if} \quad k_1, k_t \geq 0
\]

\[
\hat{2}(k_t) \quad \text{if} \quad k_2, k_t \geq 0
\]

The equilibrium skill premium is increasing in the physical capital per young household if the elasticity of the demand for children with respect to the physical capital is low enough, which would guarantee equivalence between the two expressions (6) and (7).

Finally, we have to take into account that the number of children are bounded from above if market child care is used. This is because market child services are supplied by unskilled labor, which is a fixed amount of the total number of young households (=\(\mu\)), therefore we have that,
3.4.1 Steady-State

In this section we analyze the necessary conditions for the existence and uniqueness of the steady-state. We start by checking the conditions under which we can guarantee the existence of the steady-state. Evaluating the $A(k_t)$ at $k_t = 0$,

$$
\dot{A}(0) = \max[(z=(1_i \mu_F))f(1_i b)(1_i o)(1_{1/2})] (1_{1/2}) h + b \mu g;
$$

and calculating the first derivative,

$$
\dot{A}(k_t) = \max[(z=(1_i \mu_F))b(\mu_F)g; 0 = (z=(1_i \mu_F))b(\mu_F)g > 0]
$$

and, if $k_1 \geq k_t \geq k_0$, we can say that $\dot{A}(k_t) > 0$ if the elasticity between “znit” and “kt” is low enough (i.e. when $\frac{1}{4} < (1_{1/4})$).

Since $\dot{A}(k_t)$ is a continuous function of $k_t$ (see Appendix 5); $\dot{A}(0) > 0$ and $\dot{A}(k_t) > 0$, there must exist a steady-state equilibrium if there exists $k_t$ such that $\dot{A}(k_t) < k_t$. Since $\dot{A}(k_t) = \frac{z(1_i \mu_F)}{g(k_t)}$ if $k_t > (k_0; 1)$, then we can assure that there exists $k_t$ such that $\dot{A}(k_t) < k_t$. Furthermore, there exists only
one steady-state value $k_{SS} = \frac{z(1_i \otimes b_i)}{b_2}$; if $\hat{A}(k_t) < 0$, where we may have the following three situations:

- $k_{SS} < \bar{k}^1$
- $\bar{k}^1 < k_{SS} < \bar{k}^2$
- $k_{SS} > \bar{k}^2$

Of these three possible steady-state equilibria, we are going to analyze the last two. If $k_{SS} < \bar{k}^1$ we would only observe the negative relationship between fertility and female labor supply as the economy grows.

The conditions that have to be satisfied in order to assure that $\bar{k}^1 < k_{SS} < \bar{k}^2$ can be summarized in the following expression,

$$\frac{1}{\bar{k}^2} = \frac{1}{\bar{k}^1} = \frac{1}{k_{SS}} = \frac{1}{\bar{k}^2}$$

or equivalently,

$$[\hat{\theta}(\bar{k}^1)^{\frac{1}{\bar{k}^1}} + (1 - \hat{\theta})h_{\bar{k}^1}((2 \otimes \mu_i) \otimes (2 \otimes \mu_i + \frac{1}{2}\mu))]^{\frac{1}{\bar{k}^2}} = b$$

and

$$[\hat{\theta}(z b (1 \otimes \theta) = \hat{\theta} \frac{1}{\bar{k}^1} + (1 - \hat{\theta})h_{\bar{k}^1}(2 \otimes \mu_i) \otimes (2 \otimes \mu_i + \frac{1}{2}\mu)(1 \otimes x^n))^{\frac{1}{\bar{k}^2}}]^{\frac{1}{\bar{k}^2}} = b$$
where, $0 < x^u < 1$ and $x^s(k) > 0$. The dynamic evolution for the stock of physical capital per young household, the number of children and female labor supply can be seen in Appendix 2. If we assume that the economy starts with an initial stock of physical capital per young household $k = k_0$ (as we show in the graph), then the economy will arrive at the steady-state in three periods (each period consisting of 25 years). From $k_0$ to $k_2$, fertility decreases and female labor supply increases as the economy grows (this would capture the demographic transition). In the last period (i.e. in the last 25 years), the economy moves from $k_2$ to $k_3$, fertility remains constant and female labor supply continues to increase as the economy grows20.

Now, we are going to analyze the transition dynamics when the steady-state $k_{SS} > \overline{k}^2$. In order to make sure that the steady-state value for the stock of physical capital per young household $k_{SS} > \overline{k}^2$, it must be true that,

$$\frac{1}{4}(k_{SS}; x^u = 1) > \frac{1}{4}(\overline{k}^2; x^u = 1) = \frac{1}{\gamma_2} = \frac{1}{4}(\overline{k}^1; x^u = 0)$$ (14)

or equivalently21,

$$\left[\frac{\gamma}{\gamma_2} \left(1 - \frac{\gamma}{\gamma_2} \right) \frac{\gamma_2}{\gamma} \left(2 \mu - \gamma \right) \left(1 - \frac{\gamma}{\gamma_2} \right) \right] \left(1 - \frac{\gamma}{\gamma_2} \right) = \frac{1}{\gamma_2} \left(2 \mu - \gamma \right) \left(1 - \frac{\gamma}{\gamma_2} \right)$$ (15)

where

$$\gamma_1(\overline{k}^1) = @(2 \mu + \gamma \mu)$$ (16)

Thus, in order to make sure that the steady-state value for the stock of physical capital per young household $k_{SS} > \overline{k}^2$, the parameters must satisfy the following condition:
Furthermore in order to guarantee that the upper bound imposed by the child care sector is not binding, the following restriction on the parameters must be satisfied,

$$\frac{(1_i \ b)(1_i \ \circ) h^{3/4} (z b(1_i \ \circ)) = b^{3/4} + (1_i \ \circ) h^{3/4} (2_i \ \mu)^{3/4} (1_i \ \mu)^{3/4}}{b} > \frac{1}{\sqrt{2}} \quad (17)$$

The dynamic evolution of the stock of physical capital per young household, the number of children and female labor supply can be seen in Appendix 3. We will consider that the economy reaches the steady-state in three periods (each period consisting of 25 years) starting with an initial stock of physical capital per young household of $k = k_0$. From $k_0$ to $k_2$, fertility decreases and female labor supply increases as the economy grows (this would capture the demographic transition). In the last period (i.e. in the last 25 years) the economy moves from $k_2$ to $k_3$, fertility increases and female labor supply continues to increase until the economy converges to the steady-state. Once the economy attains the steady-state, women will do not allocate any fraction of their available non-leisure time to childbearing.

### 4 Conclusions

This paper analyzes the possibility of a non-negative relationship between fertility and female labor supply as the economy’s income per capita increases. Since the Second World War, fertility and female labor supply have been negatively related in all developed countries (see, e.g., Ahn and Mira (1999), Hotz et al. (1997), Willis (1987)). This negative relationship has been explained in literature by using the theory of allocation of time developed by Mincer (1963) and Becker (1965). According to the “female cost of time hypothesis” childbearing is a time-intense activity, provided by females, and
when “women do participate in the market, the cost of time is determined by their (marginal) wage rate”. Therefore, the higher the market wage rate the higher the opportunity cost of childbearing, which induces an increase in female labor supply and may induce a decrease in fertility demand if the substitution effect dominates the income effect.

However, Ahn and Mira (1999) point out a recovery of the total fertility rates and an increasing female labor supply since the second half of the eighties in some OECD countries. In fact, the female labor supply has maintained its upward trend, with most of its increase being due to the increase of labor supply from women with newborn children. Ahn and Mira (1999), Hotz et al. (1997), among others, indicate that one possible explanation for this positive relationship between fertility and female labor supply may be the increase in the availability of market child care services. If the price of child-care services per unit of hour falls relative to the wage per unit of skilled labor, female labor supply and fertility may both increase.

We have built a general equilibrium overlapping generations model in which we have emphasized the income effect in the demand for children. In particular, we have considered an increase in the skill premium, which causes a relative decrease of the cost of childbearing with respect to the wage rate per unit of skilled labor, which induces skilled women to substitute market activities for non-market activities (childbearing) without giving up having children. Therefore, skilled women may increase their labor supply at the expense of less home production time and as the price of child-care declines relative to household income, they do not need to decrease their demand for children, purchasing childbearing time in the market.

This model is able to mimic the demographic transition experienced in the U.S., from a Beckerian phase to another phase, characterized by a non-negative relationship between fertility, household income and female labor supply. We have incorporated as the mechanism in the increase in the skill premium observed in the U.S. in the last years. In the first Beckerian phase, market child care is not used, and as long as wages increase, women work more and have less children. As market child care becomes relatively cheap it is used, and women increase their labor supply and have more children.
References


5 Appendix 1

Each household faces the following problem,

\[
\text{max} \quad u_t = \text{max} \quad f \times n_t; s_t; c_{t+1} \quad \text{g}
\]

s.t:

\[
s_t + [p_{ch} x_t + w_t^c(1 \; x_t)] z_n_t = w_t^c(2 \; \mu) + w_t^v \mu \quad \text{g}
\]

\[
c_{t+1} = (1 + r_{t+1}) s_t \\
1 \quad x_t \; \; 0; \\
(1 \; \mu \; \mu) \quad z(1 \; x_t)n_t \; \; 0
\]

The associated Lagrangian function is\textsuperscript{22},

\[
\text{g} \ln(n_t) + (1 \; \text{g}) \ln(1 + r_{t+1})(w_t^c(2 \; \mu) + w_t^v \mu \; [p_{ch} x_t + w_t^c(1 \; x_t)] z_n_t) + \quad \text{g}
\]

\[
\text{g} t x_t + o_t (1 \; x_t) + \text{g} (1 \; \mu \; \mu) \; j \; z n_t (1 \; x_t) \quad (19)
\]

The F.O.C.'s are (taking into account the control variables are \( n_t \) and \( x_t \)),

\[
\frac{\text{g}}{n_t} = \frac{(1 \; \text{g})(w_t^c(1 \; x_t) + p_{ch} x t) z}{s_t} + - t z (1 \; x_t) \quad (20)
\]

\[
\frac{(1 \; \text{g})(w_t^c(1 \; x_t) + p_{ch} x t) z}{s_t} + s_t o_t + - t z n_t = 0 \quad (21)
\]

\[
\text{g} t x_t \; \; 0; \quad x_t \; \; 0; \quad \text{g} x_t = 0 \\
\text{g} t x_t \; \; 0; \quad 1 \; x_t; \quad \text{g} x_t (1 \; x_t) = 0 \quad (22)
\]

\[
- t x_t \; \; 0; \quad (1 \; \mu \; \mu) \; z n_t (1 \; x_t); \quad - t [(1 \; \mu \; \mu) \; j \; z n_t (1 \; x_t)] = 0
\]

We may have ..ve types of solution,
The F.O.C.'s that must be satisfied for the first solution are,

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

for the second solution,

\[ zn_t(1_i x_t) = zn_t(1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

for the third solution,

\[ zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

(23)

\[
\frac{\partial}{\partial x_t} \left[ w_t^s(2_i \mu) + w_t^{u_1} \mu \right] = \frac{(1_i \mu_z)}{z} = n_t; \quad \text{where} \quad p_{ch:t}, w_t^s
\]

\[
s_t = w_t^s(1_i \mu_z) + w_t^{u_1} \mu, \quad s_t = w_t^s(1_i \mu_z) \text{ and } x_t = 0
\]

\[
\frac{\partial}{\partial x_t} \left[ w_t^s(2_i \mu) + w_t^{u_1} \mu \right] < \frac{(1_i \mu_z)}{z}; \quad \text{with} \quad p_{ch:t}, w_t^s
\]

\[
s_t = w_t^s(2_i \mu \mu z_n_t) + w_t^{u_1} \mu = \frac{(1_i \mu_z)}{w_t^s} w_t^s z_n_t, \quad x_t = 0
\]

\[
\frac{\partial}{\partial x_t} \left[ w_t^s(2_i \mu) + w_t^{u_1} \mu \right] \quad x_t = 2 (0; 1), \quad \text{and}
\]

\[
s_t = w_t^s(2_i \mu \mu z_n_t) + w_t^{u_1} \mu = \frac{(1_i \mu_z)}{w_t^s} z_n_t \quad \text{ where } w_t^s = p_{ch:t}
\]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 0; \quad zn_t(1_i x_t) = zn_t = (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ x_t = 1; \quad zn_t(1_i x_t) < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]

\[ 0 < x_t < 1; \quad zn_t(1_i x_t) = 0 < (1_i \mu_z); \quad \sigma_t = 0; \quad \tau_t = 0 \]
and for the fourth solution, we have,

\[ n_t = \frac{\otimes [w_t^s (2 \mu_i + w_t^{u_i}] + z p_{ch:t}}{z p_{ch:t} + p_{ch:t} z n_t} \quad x_t^u = 1, \quad \text{and} \]

\[ s_t = w_t^s (2 \mu_i + w_t^{u_i} - w_{w_t} + (1 - \mu_t) p_{ch:t} z n_t) \]

where \( w_t^s \), \( p_{ch:t} \)

for the fifth solution,

\[ \frac{\otimes [w_t^s (2 \mu_i + w_t^{u_i}] + z p_{ch:t} x_t}{z (1 - \mu_F)} = n_t; \quad \text{with} \quad p_{ch:t}, w_t^s \]

\[ s_t = w_t^s (1 - \mu_F) + w_t^{u_i} - w_{w_t} + (1 - \mu_F) p_{ch:t} z n_t x_t \]

\[ \frac{\otimes [w_t^s (2 \mu_i + w_t^{u_i}] + z p_{ch:t} x_t}{z (1 - \mu_F)} (1 - \mu_F) (1 - x_t) = 0 \]

The fifth type of solution would imply that females may increase their demand for children without binding the time constraint (= \( (1 - \mu_F) \)) if they buy childbearing time in the market. If the skill premium is high enough (in any case smaller than \( (1 - 3/4) \)) such that when childbearing time is not bought in the market the demand for children is smaller than the upper bound, i.e.,

\[ n_t = \frac{\otimes [(2 \mu_i + (1 - 3/4)\mu)] + z w_t^s (2 \mu_i + w_t^{u_i}] + w_t^s}{z w_t^s} \]

then the fifth type of solution will never be optimal, since we obtain the following contradiction,

\[ (1 - \mu_F) > \frac{\otimes [w_t^s (2 \mu_i + w_t^{u_i}] + z p_{ch:t} x_t}{z w_t^s} \]

\[ \frac{\otimes [w_t^s (2 \mu_i + w_t^{u_i}] + z p_{ch:t} x_t}{z (1 - \mu_F)} (1 - \mu_F) (1 - x_t) \]

But

\[ \frac{(1 - \mu_F)}{(1 - \mu_F)} \quad (1 - \mu_F) \quad 8x_t \quad 0 \]

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However, if the skill premium is too low, either the first or the fifth types of solution could be optimal. The first type of solution is associated with a lower level of savings and higher number of children. The fifth type of solution is associated with a higher level of savings and lower number of children. For a low enough \( \hat{\text{p}} \), if \( \text{w} \) the fifth type of solution will not be optimal\(^{23} \). In particular, if \( \hat{\text{p}} < \frac{\ln(s_1/s_5)}{\ln(n_1/n_5)} \), which value will depend on all prices and, therefore, on the economy’s parameters. We can assure that \( \hat{\text{p}} \) will be always positive and less than one\(^{24} \).

In graph 1 and graph 2 we have considered the possible solutions depending on the relation between the wage rate per unit of skilled labor and the market childcare price per pair of children and unit of time (i.e. \( \text{w} \) \( \text{S} \), \( \text{p} \)), and the optimal time of childbearing bought in the market (i.e. \( x_t \) \( [0;1] \)).

where,

\[
\begin{align*}
    s(w_j)_{\text{max}} &= w_j(2 - \mu) + w_j \mu = \text{maximum amount of savings if } n^a_t = 0 \text{ given by the budget constraint, with } j = a,b \text{ and } w_a < w_b; \\
    s_j &= w_j(1 - \mu) + w_j \mu = \text{maximum amount of savings if } n^a_t = (1 - \mu) = z \text{ and } x_t^a = 0; \text{ given by the budget constraint}; \\
    s_a &= w_a(2 - \mu) + w_a \mu = \text{maximum amount of savings if } n^a_t = (1 - \mu) = z \text{ and } x_t^a \in (0;1) \text{ given by the budget constraint, when } w_a < \text{p}; \\
    n_1 &= (1 - \mu) = z = \text{maximum pair of children if } x_t^a = 0; \text{ given by the time constraint}; \\
    n^a &= (w_a(2 - \mu) + w_a \mu) z = p = \text{maximum pair of children if individuals do not save at all and } x_t^a = 1; \text{ given by the budget constraint}; \\
    n_5 &= (1 - \mu) = z(1 - x_t^a) = \text{maximum pair of children if } x_t^a \in (0;1); \text{ given by the time constraint};
\end{align*}
\]

\[\hat{a} = \frac{w_a}{w_a^a} x_t^a + w_a^a = \text{the slope of the budget constraint when } w_a^a < \text{p} \text{ and } x_t^a \in (0;1);\]

\[\hat{a}^a = \hat{a}^b = z w_s = zp = \text{the slope of the budget constraint when } w_s = \text{p}.\]
Appendix 2

See graph 3, where,

\[ k_{aa} \text{ such that } \frac{1}{2}(k = k_{aa}, x = 0; zn = \frac{x}{2(2 + \mu) + \frac{1}{2} \mu}) = \frac{1}{\sqrt{2}} \]

\[ k_{bb} \text{ such that } \frac{1}{2}(k = k_{bb}, x = 1; zn = \frac{x}{2(2 + \mu) + \frac{1}{2} \mu}) = \frac{1}{\sqrt{2}} \]

\[ n_0 = \min \left( \frac{1}{z} \frac{\mu}{2} ; \frac{b}{\phi(2 + \mu) \mu} \right) \]

\[ n_1 = \frac{\mu}{z} \left[ 2(2 + \mu) + \frac{1}{2} \mu \right] \]

\[ n_2 = \frac{\mu}{z} \left[ 2(2 + \mu) + \frac{1}{2} \mu \right] x_2 \]

\[ n_3 = \frac{\mu}{z} \left[ 2(2 + \mu) + \frac{1}{2} \mu \right] x_3 \]

and female labor supply,

\[ f_{ls_0} = \frac{1}{z} zn_0, \quad f_{ls_1} = \frac{1}{z} zn_1; \]

\[ f_{ls_2} = \frac{1}{z} \left[ 2(2 + \mu) + \frac{1}{2} \mu \right] x_2, \quad f_{ls_3} = \frac{1}{z} \left[ 2(2 + \mu) + \frac{1}{2} \mu \right] x_3 \]
Appendix 3

See graph 4, where,

\[ k_{aa} \text{ such that } \frac{1}{4}(k = k_{aa}; x = 0; zn = \Theta[(2 \mu) + \frac{1}{2}\mu]) = \frac{1}{\sqrt{2}} \]

\[ k_{bb} \text{ such that } \frac{1}{4}(k = k_{bb}; x = 1; zn = \Theta[(2 \mu) + \frac{1}{2}\mu]) = \frac{1}{\sqrt{2}} \]

\[ n_0 = \min\left(\frac{1}{z} \frac{\mu_x f(2 \mu + \frac{b}{w_0(k_0)} \mu_g)}{z} + \frac{b}{w_0(k_0)} \mu_g\right) \quad n_1 = \frac{\Theta}{z}[(2 \mu) + \frac{1}{2}\mu] \]

\[ n_2 = \Theta[(2 \mu) + \frac{1}{2}\mu]x_2 \ldots, n_3 = \Theta[k_3] \]

and female labor supply,

\[ f_{ls_0} = 1 \mu zn_0, \quad f_{ls_1} = 1 \mu zn_1; \]

and,

\[ f_{ls_2} = 1 \mu [(2 \mu) + \frac{1}{2}\mu]x_2, \quad f_{ls_3} = 1 \]
8 Appendix 4

The stock of physical capital at period $t+1$ will be given by the savings of the young population at the end of period $t$,

$$K_{t+1} = s_t L_t$$

rewriting that condition in terms of the number of young households in period $t$,

$$k_{t+1}n_t = s_t$$

Taking into account households' optimal choices of the demand for children, market use of child care and savings we can obtain the physical capital per young household at time $t+1$ as

$$k_{t+1} = \frac{s_t}{n_t} = \frac{s_t}{(24)}$$

In the former expression, $w_t^e$ depends on $k_t$ and $l_t^e$, where $l_t^e$ depends on $x(k_t)$ and $n_t$, being $n_t$ a function of $w_t^e$. In order to obtain a function of $n_t$ depending only on $k_t$; we use expressions (1), (3), (4), and (8):

$$z(1_i \mu_F)[w_t^e(1_i \mu_M) + w_t^i \mu]$$

if $\frac{1}{4} < (1\equiv\frac{1}{4})$ and $z n_t = (1_i \mu_F)$

$z(1_i \mu_M) \equiv 0$;

if $\frac{1}{4} < (1\equiv\frac{1}{4})$ and $z n_t < (1_i \mu_F)$

$z(1_i \mu_M) \equiv 0$;

if $\frac{1}{4} > (1\equiv\frac{1}{4})$.
\[
\text{minf}(1_i \mu_k; \text{zt}(2_i \mu) + \frac{\text{zt}(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g}{(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g})^{2_i \mu_i}} = 8
\]

\[
\text{zt}(k_t) = \text{zt}(2_i \mu) + \frac{\text{zt}(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g}{(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g})^{2_i \mu_i}
\]

\[
\text{zt}_{t_1}(k_t) = \text{zt}(2_i \mu) + \frac{\text{zt}(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g}{(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g})^{2_i \mu_i}
\]

\[
\text{zt}_{t_1} = \begin{cases} 0 & \text{if } \frac{1}{4} < (1\frac{1}{4}) \\ 2 (0; 1) & \text{if } \frac{1}{4} = (1\frac{1}{4}) \\ 1 & \text{if } \frac{1}{4} > (1\frac{1}{4}) \end{cases}
\]

Or in an equivalent form:

\[
\text{zt}_{t_1} = \begin{cases} 0 & \text{if } \frac{1}{4} < (1\frac{1}{4}) \\ \text{zt}(2_i \mu) + \frac{\text{zt}(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g}{(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g})^{2_i \mu_i} & \text{if } \frac{1}{4} = (1\frac{1}{4}) \\ \text{zt}(2_i \mu) + \frac{\text{zt}(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g}{(1_i: b(1_i) h^{2}(2_i \mu i \text{zt})^{2_i \mu_i} g})^{2_i \mu_i} & \text{if } \frac{1}{4} > (1\frac{1}{4}) \end{cases}
\]
Furthermore, we can show that,

\[
\frac{\partial (z_{nt}; k_t)}{\partial n_t} = 8 \begin{cases} 
  z > 0; & 8k_t, 0 \quad \text{if } \frac{1}{4} < (1-\frac{1}{2}) \\
  & \text{& } z_{nt} = (1 \text{ i } \mu_F) \\
  \text{z i } \frac{\partial v^t_{max}}{\partial n_t} > 0; & 8k_t, 0 \quad \text{if } \frac{1}{4} < (1-\frac{1}{2}) \\
  & \text{& } z_{nt} < (1 \text{ i } \mu_F) \\
  = z > 0; & 8k_t, 0 \quad \text{if } \frac{1}{4}, (1-\frac{1}{2}) \\
\end{cases}
\]

and:

\[
\frac{\partial (z_{nt}; k_t)}{\partial k_t} = 8 \begin{cases} 
  > 0; & \text{if } \frac{1}{4} < (1-\frac{1}{2}) \\
  & \text{& } z_{nt} = (1 \text{ i } \mu_F) \\
  = 0 & \text{if } \frac{1}{4} = (1-\frac{1}{2}) \\
  < 0 & \text{if } \frac{1}{4} > (1-\frac{1}{2}) \\
\end{cases}
\]

Therefore, we can apply the implicit function theorem to obtain a relationship between the number of couple of children and the level stock of physical capital,

\[
z_{nt} = \begin{cases} 
  (1 \text{ i } \mu_F) & \text{such that } 1(k_t) < 0; \text{ if } \frac{1}{4} < (1-\frac{1}{2}) \\
  & \text{& } z_{nt} = (1 \text{ i } \mu_F) \\
  (2 \text{ i } \mu + \frac{1}{2} \mu) & \text{if } \frac{1}{4} = (1-\frac{1}{2}) \\
  & \text{& } z_{nt} < (1 \text{ i } \mu_F) \\
  \end{cases}
\]

Consequently, we can obtain the law of motion of the capital per young household:

\[
k_{t+1} = \frac{s_t}{n_t} = \]

31
such as,

\[ k_{t+1} = A(k_t) = \]

\[ \frac{z}{[1; \mu]} f(1_i b)(1_i \circ h)^\frac{1}{2}[0 k_t^2 + (1_i \circ) (h(1_i \mu_M))^{\frac{1}{2}}(1_i \mu_M)^{\frac{1}{2}} + b_g g] \quad \text{if} \quad k^0, k_t, 0 \]

\[ \frac{z [1_i \circ b]}{[1_i \circ b]} h^\frac{1}{2}(1_i b)(1_i \circ (1_i \circ) (h(1_i \mu_M))^{\frac{1}{2}}(1_i \mu_M)^{\frac{1}{2}} + b_g g) \quad \text{if} \quad k^1, k_t, k^0 \]

\[ \frac{z [1_i \circ b]}{[1_i \circ b]} h^\frac{1}{2}(1_i b)(1_i \circ (1_i \circ) (h(1_i \mu_M))^{\frac{1}{2}}(1_i \mu_M)^{\frac{1}{2}} + b_g g) \quad \text{if} \quad k^2, k_t, k^1 \]

\[ \frac{z [1_i \circ b]}{[1_i \circ b]} h^\frac{1}{2}(1_i b)(1_i \circ (1_i \circ) (h(1_i \mu_M))^{\frac{1}{2}}(1_i \mu_M)^{\frac{1}{2}} + b_g g) \quad \text{if} \quad k_t, k^2 \]

\[ k^0 \] is the maximum value for \( k_t \), such that the skill premium is smaller than \((1=\frac{1}{2})\), \( z_n^e = (1_i \mu_F) \) and the equilibrium value for the use of market child care is \( x_t^e = 0 \). Therefore \( k^0 \),

\[ z_n^e = \tau_1(k^0) = f(2_i \mu) + \frac{w_i \mu}{w^*(k^0; 1_i = (1_i \mu_M))} g = (1_i \mu_F) \]

or equivalently,

\[ (1_i \mu_F) = f(2_i \mu) + (b = (1_i \mu)(1_i \circ) h^\frac{1}{2}) \sqrt{[0 (k^0)^{\frac{1}{2}} + (1_i \circ) (h(1_i \mu_M))^{\frac{1}{2}}(1_i \mu_M)^{\frac{1}{2}} + b_g g} = \tau_1(k^0) \]

\[ k^1 \] is the minimum value for \( k_t \) such that the skill premium equals \((1=\frac{1}{2})\) and \( x_t^e = 0 \).

On the other hand, taking into account that the skill premium increases with \( k_t \) and decreases with \( x_t \); there exist pairs of \((k_t; x_t)\) such that both \( k_t \)
and $x_t$ increase, while the value of the skill premium remains constant at $(1=\frac{1}{2})$. $k^2$ is the minimum value for $k_t$ such that the skill premium equals $(1=\frac{1}{2})$ and $x_t = 1$.

Therefore,

$$w^s(k_t; x(k_t)) \frac{b}{b} = \frac{1}{k_t} = 1 \text{ if } k^2, k_t, k^1 \quad (33)$$

and consequently, if $k^2, k_t, k^3$;

$$A(k_t) = \frac{z(1 \circ \circ b)}{b} \quad (34)$$
9 Appendix 5

The function $\hat{A}(k_t)$ is continuous in each piecewise. Therefore, to complete the proof of its continuity, we only have to show that the expressions in the corresponding extreme points coincide.

At $\overline{K^0}$:

By definition, $k^0_t$ is a value of $k_t$ such that: $n^u(k^0) = \frac{(1 + i \mu_F)}{z}$ and $x_t(k^0) = 0$:

$$
\frac{z}{(1 + i \mu_F)} \left[ \left( 1 \right) \left( 1 + i \right) \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \right] = \frac{n^o(1 + i \mu)}{z} = \frac{n^o(1 + i \mu)}{z}
$$

Therefore, the equality is satisfied at $\overline{K^0}$.

At $\overline{K^1}$:

$$
\frac{z(1 + i \mu)}{\overline{K^1}} \left[ \left( 1 \right) \left( 1 + i \right) \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \frac{(1 + i \mu)}{1 - \mu_f(1 + i \mu)} \right] = \frac{n^o(1 + i \mu)}{z} = \frac{n^o(1 + i \mu)}{z}
$$

By definition $k^1_t$ is a value of $k_t$ such that: $\frac{l^s(k^1)}{l^u} = \frac{1}{2}$ and $x_t(k^1) = 0$: 
Consequently,
\[ \ddot{r}(k) = \circ(2i\mu) + \circ\frac{1}{s}(k) = \circ(2i\mu) + \circ\frac{1}{s}(k) \]

Therefore, the LHS can be expressed as follows:
\[
\frac{z(1i\circ)(1i)b(1i\circ)h^{\circ\frac{n}{2}}(k)}{[2i\mu \circ(2i\mu) + \circ\frac{1}{s}(k)]_{1i\circ}^{\circ\frac{n}{2}}}\
\]

Furthermore, since \( x_t(k^2) = 0 \); the RHS will be expressed as follows:
\[
\frac{z(1i\circ)(1i)b(1i\circ)h^{\circ\frac{n}{2}}(k)}{[2i\mu \circ(2i\mu) + \circ\frac{1}{s}(k)]_{1i\circ}^{\circ\frac{n}{2}}}\
\]

Therefore, the equality is satisfied at \( k^1 \):
At \( k^2 \):
\[
\frac{z(1i\circ)(1i)b(1i\circ)h^{\circ\frac{n}{2}}(k)}{[2i\mu \circ(2i\mu) + \circ\frac{1}{s}(k)]_{1i\circ}^{\circ\frac{n}{2}}} = \frac{z(1i\circ)\circ\frac{1}{s}(k)}{\circ\frac{1}{s}(k)}
\]

By definition \( k^2 \) is the value for \( k^1 \) such that: \( \frac{! s(k^2)}{! u} = \frac{1}{\circ\frac{1}{s}(k^2)} = \frac{1}{\circ\frac{1}{s}(k^2)} \). Therefore, \( ! s(k^2) = \frac{1}{\circ\frac{1}{s}(k^2)} = \frac{1}{\circ\frac{1}{s}(k^2)} \); and the LHS of the former expression is:
\[
\frac{z(1i\circ)\circ\frac{1}{s}(k)}{\circ\frac{1}{s}(k)} = \frac{z(1i\circ)\circ\frac{1}{s}(k)}{\circ\frac{1}{s}(k)}
\]
Graph 1: BUDGET CONSTRAINT with $P_{ch} > \omega^s$

$s((\alpha_s)^{max})$

$s((\alpha_s)^{max})$

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Graph 2: BUDGET CONSTRAINT with $P_{ch} \leq \omega$
\[ K_{t+1} = K_t \]

\[ X = 0 \]

\[ X = 1 \]

\[ n_0 = n_1 = n_2 \]

\[ K_{t+1} = K_t = K_{t+1} = K_t = K_{t+1} = K_t \]

GRAPH 3

\[ X = 0 \]

\[ X = 1 \]

\[ n_0 \]

\[ n_1 \]

\[ n_2 \]

\[ f(t) \]

\[ f(t) \]

\[ f(t) \]

\[ f(t) \]
$K_{t+1}^k = K_t^k$ to $K_{t+1}^{k+1} = K_t^{k+1}$

$K_{t+1} = K_t$

$\text{GRAPH 4}$

$X = 0 \rightarrow K_{t+1} = K_t$

$X = 1$

$X = 0 \rightarrow X = 1$

$X = 1$

$X = 0 \rightarrow X = 1$

$X = 1$

$X = 0 \rightarrow X = 1$

$X = 1$

$X = 0 \rightarrow X = 1$

$X = 1$
10 Endnotes

1 Willis (1987) asserts that “the cost of time hypothesis suggests that the fertility demand function should distinguish between male and female wages”.

2 The child care industry grew by 106% during the 1977-1992 period, the number of paid employees in that industry grew by 146% in the same period (Source: U.S. Census Bureau). On the other hand, some papers (Blau and Robins (1988, 1989, 1991), Ribar (1992), among others) report the relative decline of the market childbearing price relative to the mean wage.

3 Child care workers’ wage rates are, on average, similar to wage rates for unskilled labor in the period 1972-1996 (Source: U.S. Labstat).

4 We thank Antonia Díaz for providing us with this data set.

5 Organized child care facilities: Day/group care center and nursery school/preschool.

6 The model is so simple that we consider childbearing time as the only cost of having children. We not consider additional costs in goods.

7 See Conesa (1999) for an analysis of how incomplete markets and idiosyncratic uncertainty may affect the optimal timing of childbearing decisions.

8 Galor and Weil (1996) consider differences in factor endowment of labor input between men and women such that the opportunity cost of raising children is higher for a man than for a woman, which gives the division of labor endogenously. However, we take the division of labor as given, assuming that only women may allocate their time to childbearing activities. Alternatively, we could have considered an exogenous income instead of exogenous men’s income (men are not necessary) but in that case we would need to make assumptions about the source of that exogenous income and its dynamics.

9 Individuals, however, live three periods: as children, as workers and as retired people.

10 The only household activity considered is childbearing. Since we are interested in analyzing the effects of increases in the skill premium, as the economy grows, on women’s fertility and labor supply decisions, only skilled time can be chosen to be allocated between market and childbearing activities. On the other hand, following Willis (1987), we consider separately male and female wages, but in a different way to Galor and Weil (1996).

11 With no loss of generality we could consider $\mu_F = 0$: However, either men or women should be endowed with unskilled labor, since the child-care market only hires unskilled labor.

12 As in Galor and Weil (1996), since the utility function is log-linear, the
addition of the logarithmic utility that the household derives from consumption in the first period of life would not affect the dynamics.

13 As in Galor and Weil (1996), since households are formed by the couple (and the children they have), to avoid the problem of matching individuals we assume that “\( n_t \)” measures the number of couples of children. Obviously, “\( n_t \)” is a continuous variable and if “\( n_t = 1=2 \)” the household has chosen to have one child.

14 See Appendix 1 for the solution of the household’s problem. We have assumed that the preference for children is not so high as to demand more children than there is time available, which would imply the use of market childcare even if its price is extremely high.

15 The wage rate that child care workers receive is, on average, similar to the wage rate per unit of unskilled labor for 1972-1996 in the U.S.A. based on data from LABSTAT (CPS).

16 Krusell et al. (1994) consider a more general specification for the production function exhibiting capital-skill complementarity that can account for nearly all the variation in the skill premium over the last 30 years in the U.S.A.

17 The relation \( x_t = x(k_t) \) is uniquely determined by the condition that \( \frac{\partial^2 x_t}{\partial k_t^2} = \frac{1}{b} \) be satisfied.

18 If the skill premium is smaller than \( (1/\lambda) \), \( \hat{A}(k_t) \) is a linear function of the skill premium and, consequently, the conditions under which we can guarantee that \( \hat{A}(k_t) \) is an increasing function in \( k_t \) are the same as those that guarantee that the equilibrium’s skill premium is an increasing function in \( k_t \):

19 If \( \hat{A}(k_t) > 0 \) then, as in Galor and Weil (1996), there would exist multiple stationary equilibria, two stable (one associated with a low income per capita, low female labor supply and high fertility and the other one associated with a high income per capita, low fertility and high female labor supply) and one unstable.

20 Female labor supply is increasing because women are buying market childbearing time \((8k_t \cdot k_{aa})\) and the use of market childbearing is increasing in \( k_t \) \((k_3 = k_{SS} > k_2)\).

21 We are assuming that, \( \frac{(1_i \cdot \partial(2_i \cdot \mu_i) \cdot (1_i \cdot \mu_M)}{\partial u} \cdot \frac{1}{2} \) in order to guarantee that the female time constraint is satisfied, therefore,

\[
(2_i \cdot \mu_i \cdot (1(R^2)) = (2_i \cdot \mu_i \cdot (\partial(2_i \cdot \mu) + \partial/\mu)) \cdot (1_i \cdot \mu_M)
\]
This is the same condition we imposed in order to get a demographic transition with a decline in fertility (see footnote (10)).

Because the utility function is logarithmic in \( n_t \), “\( n_t = 0 \)” will never be optimal. Therefore, \( z_n(1 - x_t) = 0 \) will never be an optimal choice unless \( x_t = 1 \), which is already considered in the Lagrangian function.

A sufficient condition for the third type of solution not to be an optimal solution is the following:

\[
\frac{\mu}{(1 + b)(1 - \sigma)^{1-\sigma}} = (1 + \mu_e)
\]

This condition guarantees that, \( \Phi((2 + \mu) + (w^u - w^s)\mu) = (1 + \mu_e) \) for \( k = 0 \), which assures that if \( p_{cht} \), \( w^s \) the optimal household choice for \( x_t = 0 \). Such a condition is satisfied when \( \mu_e > 1 \):

If we represent household’s optimal choices for the first solution as \( (n_1; s_1) \) and for the fifth solution \( (n_5; s_5) \), then we can say that \( \frac{n_1}{n_5} < 1 \) and \( \frac{s_1}{s_5} > 1 \):

Take into account that the subscripts in the variables denote “time”, and the superscripts do not denote time but a particular value for that variable that holds a given condition. \( (k^0 > 0 \text{ if } \Phi((2 + \mu) + (w^u - w^s)\mu) > (1 + \mu_e)) \).

As long as \( k_2 \), \( k_3 \), \( k_1 \) there exists a value for \( x_t = 2 \) (increasing in \( k_t \)) for which \( \frac{w}{w^s} = \frac{1}{\frac{1}{2}} \) and, therefore, the household optimal decision is consistent. Finally, if \( k_t = k^2 \), skill premium remains equal to \( 1 = \frac{1}{2} \text{ i.e. } \frac{w}{w^s} = \frac{1}{\frac{1}{2}} \), the equilibrium value for the use of market child care is \( x_t^n = 1 \) and \( \Phi((2 + \mu) + \frac{1}{2}\mu) = \Phi((2 + \mu) + \frac{1}{2}\mu) \):