Multiple equilibria, fiscal policy, and human capital accumulation*

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Abstract

This paper shows that in a three-sector model of endogenous growth with physical and human capital accumulation, taxation policy can generate indeterminacy under plausible parameterizations. The key for this result is that the accumulation of human capital is a non-market activity in which individuals combine their non-working time with intermediate goods that are provided by the market. This assumption is consistent with the microfoundations of human capital accumulation found by the literature on life-cycle earnings.

Key Words: indeterminacy; human capital investment; economic growth; fiscal policy.

JEL Classification Numbers: E32; E62; O41

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1. Introduction

In recent years, there has been an extensive literature that examines the existence of indeterminate equilibria in dynamic general equilibrium models.\textsuperscript{1} The motivation of this search is that indeterminacy is able to account for endogenous business fluctuations and for some empirically observed growth episodes that cannot be reconciled with more traditional literature.\textsuperscript{2} The possibility of a continuum of equilibria is the consequence of some market imperfections. Usually, the imperfections are assumed to come from external effects in production or monopolistic competition. Furthermore, it has been showed that indeterminacy appears under empirically realistic assumptions.

The intent of this paper is to show that multiple equilibria can also arise straightforwardly without abandoning the hypothesis of perfect markets and constant returns in production. We argue that the market imperfections generated by the taxation policy can generate indeterminacy in a three-sector endogenous growth model with physical and human capital accumulation. As a crucial ingredient for our result, we depart from the traditional endogenous growth model with human capital accumulation by making deeper the microfoundations of the process of human capital accumulation. Building on Heckman (1976), we specify the accumulation of human capital as a non-market activity that uses effective labor and goods (and services) acquired in the market (intermediate goods, henceforth) as inputs. Therefore, the investment in human capital has a direct component given by the purchases of intermediate goods and an indirect component given by the opportunity cost of the time spent in accumulating human capital. While the former component is perfect substitute of physical capital investment and consumption, the later component is no perfect substitute of those alternative allocations of income. Both components are empirically relevant. For

\textsuperscript{1} See Benhabib and Farmer (1999) for a complete survey of this literature.

\textsuperscript{2} See Benhabib and Galí (1995) for an overview of the empirical predictions of growth models with multiple equilibria.
instance, Perli and Sakellaris (1998) estimate the total educational expenditures and the foregone labor income of students in US at 8% and 3.6% of GNP, respectively.

Traditionally, the endogenous growth models with human capital accumulation have often alternatively considered only one of these two components of human capital investment. Some models assume that human capital accumulation is a pure non-market activity (see, e.g., Lucas, 1988). Other models instead consider that human capital accumulation is a pure market activity where individuals increase their human capital by directly acquiring the intermediate goods without any other manipulation (see, e.g., Rebelo and Stokey, 1995). By contrast, Jones, Manuelli and Rossi (1993) also specified a process of human capital accumulation like ours. However, they postulate a two-sector structure in which human capital accumulation requires market consumption goods and effective labor. We instead consider a third sector whereby intermediate goods are first produced with a specific technology that uses physical capital and effective labor as inputs. These intermediate goods are then used exclusively, together with effective labor, for the accumulation of human capital.

We complete the model with the introduction of a tax on physical capital income, a tax on labor income, a subsidy to the purchase of intermediate goods and a subsidy to the opportunity cost of time spent by individuals in manipulating these intermediate goods. We will prove that the fiscal policy mix determines the equilibrium dynamics. In this kind of models, equilibrium prices are determined independently of quantities and, moreover, the stability properties of the price process and of the quantity process are dictated by the ranking of factor intensities across sectors.\(^3\) In absence of fiscal policy, the economic system is saddle-path stable since an unstable adjustment process in prices or quantities is offset by adjustment in the other variable. However, the distortion from the taxation policy introduces a wedge between the factor intensity ranking in terms of factor cost shares, which governs the price behavior, and the ranking in terms of factor quantity ratios,

\(^3\) The stability properties of those processes are established by calling the Stolper-Samuelson effect and the Rybczynski effect, respectively. The role of the factor intensity ranking in the transitional dynamics is extensively presented in Bond, Wang and Yip (1996).
which determines the behavior of production. When this distortion is sufficiently high, the previous two rankings of factor intensities are no consistent and, thus, the usual saddle-path stability is lost. In particular, under some plausible combinations of the fiscal instruments, the equilibrium is locally indeterminate, that is, there is a continuum of equilibrium paths converging to a unique balanced growth path. Therefore, knowledge of the initial stocks of physical and human capital is no longer sufficient to pin down the equilibrium path followed by the economy. The latter path is fully determined by the initial choices of consumption and allocation of time, which are driven by self-fulfilling prophecies.

A similar result was found by Bond, Wang and Yip (1996) in a two-sector model where the accumulation of human capital is a pure market activity. However, we will see that indeterminacy emerge in our model under conditions for the reversal in the factor intensity ranking that are opposite of those found by them. Moreover, we derive indeterminacy under a different fiscal policy structure. The aforementioned authors consider sector-specific fiscal policies in which labor income from the sector producing new human capital is not taxed. They need this assumption to address indirectly the “non market” aspect of the human capital accumulation process. By contrast, given our three-sector production structure, we will be able to obtain multiplicity with a uniform fiscal treatment of each factor across the two market sectors.

The paper is organized as follows. In Section 2 we present the model and define the competitive equilibrium of the economy. Section 3 characterizes the equilibrium dynamics, analyzing the conditions for the multiplicity of equilibrium paths. Section 4 briefly discusses the economic mechanism underlying the possibility of multiple equilibria and touches the plausibility and the empirical implications of the stability results. Some lengthy proofs are contained in the appendix.

2. The Model
Let us consider an economy populated by a continuum of identical, everlasting households. For convenience, we assume that population growth is zero and normalize population size to one. The households’ preferences are represented by

$$U = \int_0^\infty \left( \frac{C(t)^{1-\sigma} - 1}{1 - \sigma} \right) e^{-\rho t} dt,$$

where $C(t)$ is consumption, $\rho > 0$ is the constant subjective rate of time preferences, and $\sigma > 0$ denotes the inverse of the constant elasticity of inter-temporal substitution. Each household is endowed with a unit of time that is allocated to labor, and to accumulating human capital. If $l(t)$ denotes the fraction of time devoted to labor, then the effective labor supplied by the household is $l(t)h(t)$, where $h(t)$ is the per capita stock of human capital.

There are three productive sectors in this economy. The first one produces a homogenous physical good, $Y(t)$, which can be either consumed or added to the stock of physical capital, $K(t)$. This physical good is produced according to the Cobb-Douglas technology

$$Y(t) = A[v(t)K(t)]^\beta [u(t)h(t)]^{1-\beta} = Au(t)h(t)z_1(t)^\beta,$$

where $A > 0$ is the scale factor, $v(t)$ and $u(t)$ are respectively the shares of physical capital and human capital allocated to the physical goods sector, and $z_1(t) = [v(t)K(t)][u(t)h(t)]$. The second sector produces an intermediate good, $E(t)$, which is acquired by households to increase their stock of human capital. In this sector, we also posit a Cobb-Douglas production function, with

$$E(t) = B[(1 - v(t))K(t)]^\alpha ((l(t) - u(t))h(t))^{1-\alpha} = B(l(t) - u(t))h(t)z_2(t)^\alpha,$$

where $B > 0$ is the scale factor, and $z_2(t) = [(1 - v(t))K(t)][(l(t) - u(t))h(t)]$. Finally, the third sector involves non-market activities resulting in increases of households’ stock of human capital, which enhances the efficiency units of their labor supply. We assume that households need to spend
time in manipulating the intermediate goods so as to increase their stock of human capital. 4 More precisely, we postulate a Cobb-Douglas technology of human capital production. Hence, the law of motion of human capital stock is

$$\dot{h}(t) = \eta I_h(t)^{\theta} \left( (1 - l(t))h(t) \right)^{1-\theta} - \eta h(t),$$

(4)

where $I_h$ represents the gross investment demand for intermediate goods from each household, and $\eta \geq 0$ is the constant rate of depreciation of human capital stock.

The government in this economy sets a flat-rate tax, $t_k$, on physical capital income, a flat-rate tax, $t_h$, on labor income, a flat-rate subsidy, $s_E$, to the purchases of intermediate goods, and a flat-rate subsidy, $s_w$, to the before-tax opportunity cost of accumulating human capital. We assume that the government faces a balanced budget constraint in each moment in time, that is,

$$T(t) = t_h r(t) K(t) + t_h w(t)l(t)h(t) - s_E p(t) I_h(t) - s_w w(t)(1 - l(t))h(t),$$

(5)

where $T(t)$ is either a lump-sum tax or a lump-sum transfer depending on the sign of the right hand side of (5). The budget constraint of a household is thus

$$(1 - t_h) r(t) K(t) + (1 - t_h) w(t)l(t)h(t) + s_w w(t)(1 - l(t))h(t)$$

$$= C(t) + \dot{K}(t) + (1 - s_E) p(t) I_h(t) + T(t) - \delta k(t),$$

(6)

where $r(t)$ and $w(t)$ are respectively the rental rate of physical capital and the wage rate, $p(t)$ is the relative price of intermediate goods in units of physical goods, and $\delta \geq 0$ is the constant rate of depreciation of physical capital stock.

Given the initial stocks of physical and human capital, which we will respectively denotes by $K_0$ and $h_0$, and fiscal policy $\{t_k, t_h, s_E, s_w\}$, a competitive equilibrium under balanced budget is

4 The same assumption may be made for the accumulation of physical capital and consumption. We omit this possibility to facilitate the exposition and the understanding of our results and conclusions.
defined as a set of path for prices \( \{r(t), w(t), p(t)\} \), allocations \( \{C(t), l(t), v(t), u(t), I_h(t), T(t)\} \), and both capital stocks \( \{K(t), h(t)\} \), such that

(i) the representative household’s choice of the paths \( \{C(t), l(t), I_h(t), K(t), h(t)\} \) maximizes (1) subject to (6), (4) and to non-negativity constraints on all variables;

(ii) the paths \( \{v(t), u(t), K(t), h(t)\} \) maximize firms’ profits;

(iii) the government obeys its inter-temporal budget constraint (5); and

(iv) the market-clearing conditions for physical goods and for intermediate goods hold, i.e.,

\[
Y(t) = C(t) + \dot{K}(t) - \delta K(t),
\]

\[
E(t) = I_h(t).
\]

In equilibrium, competition among profit-maximizing firms ensures that both factors are paid their marginal products. Hence, if both physical capital and human capital are perfectly mobile across sectors, the profit maximization conditions are

\[
r(t) = \beta A z(t)_{\beta} = p(t) \alpha B z_{\beta} (t)^{\alpha-1},
\]

\[
w(t) = (1 - \beta) A z(t)_{\beta} = p(t) (1 - \alpha) B z_{\beta} (t)^{\alpha}.
\]

The representative household’s problem involves three margins. First, total income must be allocated between consumption of physical goods and total investments. This implies that the marginal substitution between consumption at different dates must equalize its corresponding relative price, i.e.,

\[
e^{-\rho t} \frac{C(t)}{C(0)} = e^{-R(0,t)},
\]

where \( R(0, t) = \int_0^t [(1 - t_{\alpha}) r(s) - \delta] ds \) is the cumulative rate of return between the initial period and time \( t \).
Secondly, income allocated to total investments must be efficiently distributed between the accumulation of physical capital and the purchase of intermediate goods. The optimal portfolio selection is given by the equality between the marginal return of a unit invested in the accumulation of each capital. Thus, since the present value of the cost and the benefit of investing in physical capital is the unit, the portfolio selection is given by the following non-arbitrage condition:

\[
(1 - s_E) p(t) = \Theta \gamma I_h (t)^{\beta - 1} \left( (1 - l(t)) h(t) \right)^{1 - \Theta} V (t), \tag{11}
\]

where \( V(t) \) is the discounted marginal value of the household's human capital stock at time \( t \), i.e.,

\[
V(t) = \int_0^\infty e^{G(t, \tau) - R(t, \tau)} \left[ (1 - t_h) l(\tau) + s_w (1 - l(\tau)) \right] w(\tau) d\tau, \tag{12}
\]

where \( G(t, \tau) = \int_0^\tau \left[ G_l (s)^0 (1 - l(s))^{1 - \Theta} h(s)^{-\Theta} - \eta \right] ds \) is the cumulative variation of the human capital stock between time \( t \) and \( \tau \) caused by a marginal change in \( h(t) \), and \( R(t, \tau) \) is the cumulative rate of return between time \( t \) and \( \tau \).

Finally, the endowment of time must be allocated between labor supply and human capital accumulation. Optimally means that the household gets the same marginal revenue in each of them, i.e.,

\[
(1 - t_h - s_w) w(t) = (1 - \Theta) \gamma I_h (t)^{\beta} \left( (1 - l(t)) h(t) \right)^{-\Theta} V (t). \tag{13}
\]

The competitive equilibrium is then fully characterized by Eqs. (4), (5), (6), (7), (8), (9), (10), (11) and (13), the usual transversality conditions and the initial values \( K_0 \) and \( h_0 \).

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Note that the discounted value of a household's total income from human capital at any time \( t \) is equal to

\[
W(t) = \int_0^\infty e^{-\lambda(t, \tau)} \left[ (1 - t_h) l(\tau) + s_w (1 - l(\tau)) \right] w(\tau) h(\tau) d\tau.
\]

The income generated by the human capital stock at any time \( \tau \) is twofold. First, the fraction of time in efficiency units of labor allocated to working obtains an after-tax labor income. Second, the fraction of time allocated to accumulating human capital also obtains a subsidy per unit of foregone labor income. Using the law of motion of human capital (4), Expression (12) is easily obtained from the definition of \( W(t) \) .
For notational convenience, we index the fundamentals of the model economy by the vector of parameters \( \pi \). We then define \( \pi \equiv (A, B, \gamma, \rho, \sigma, \delta, \eta, \alpha, \beta, \theta, t_k, t_h, s_k, s_w) \), and \( \pi \in \Pi \), where \( \Pi = \mathbb{R}_+^3 \times \mathbb{R}_+^2 \times (0,1)^3 \times [0,1]^4 \).

3. The Equilibrium Dynamics

In the environment described in the previous section, the dynamic behavior of the economic variables crucially depends on the fiscal policy. Under some combinations of fiscal instruments, the equilibrium dynamics may be not defined by the usual property of saddle-path stability. In this section, we will characterize the dynamic behavior of the economy along the equilibrium. For that purpose, we first construct the dynamic system that fully describes the competitive equilibrium paths. As a necessary step, we reduce the relevant explanatory variables of the model. Assuming that both sectors are active, one can explicitly solve for the factor intensities in each sector, the relative price and shares of both types of capitals in each technology as functions of \( K(t), h(t) \) and \( V(t) \).\(^7\) First, we get from (9) and (11) that

\[
\begin{align*}
    z_1(t) \equiv z_1(V(t)) &= \left( \frac{\beta}{1-\beta} \right) \frac{\alpha}{1-\alpha} \Omega V(t)^{\beta-\alpha}, \\
    z_2(t) \equiv z_2(V(t)) &= \left( \frac{\alpha}{1-\alpha} \right) \Omega V(t)^{\beta-\alpha}, \\
    p(t) \equiv p(V(t)) &= \Omega V(t)^{\beta-\alpha},
\end{align*}
\]

where

\(^6\) These conditions are standard and impose that the present discounted value of both capital stocks tends to zero at the infinity.

\(^7\) See the appendix for a detailed derivation of these expressions.
\[
\Phi = \left[ \left( \frac{\beta A}{\alpha B} \right) \frac{\alpha}{1-\alpha} \left( \frac{\beta}{1-\beta} \right)^{\beta-1} \right]^{\gamma_\alpha - \beta},
\]
\[
\Omega = \left[ \left( \frac{\gamma \theta}{1-s_k} \right)^{B\Psi} \left( \frac{\alpha \phi}{1-\alpha} \right) \right]^{\gamma_\alpha - \beta - \alpha_\theta},
\]
\[
\Psi = \left( \frac{\theta(1-\alpha)}{1-\theta} \right) \frac{1-t_h-s_s}{1-s_k}.
\]

Moreover, manipulating Conditions (11) and (13) with Eqs. (8), (9) and (14), we also get
\[
I(t) \equiv I(K(t)/h(t), V(t)) = \frac{\left[ K(t)/h(t) \right] + \Psi \left[ z_1(t) - z_2(t) \right]}{z_1(t) + \Psi \left[ z_1(t) - z_2(t) \right]}, \quad (15a)
\]
\[
u(t) \equiv \nu(K(t)/h(t), V(t)) = \left( \frac{z_2(t)}{\left[ K(t)/h(t) \right]} \right) \frac{\left[ K(t)/h(t) \right] + \Psi \left[ (K(t)/h(t)) - z_2(t) \right]}{z_1(t) + \Psi \left[ z_1(t) - z_2(t) \right]}, \quad (15b)
\]
\[
u(t) \equiv \nu(K(t)/h(t), V(t)) = \left( \frac{z_2(t)}{\left[ K(t)/h(t) \right]} \right) \frac{\left[ (K(t)/h(t)) - z_2(t) \right]}{z_1(t) + \Psi \left[ z_1(t) - z_2(t) \right]}, \quad (15c)
\]

At this point, we can fully characterize the dynamic behavior of the economy along an interior equilibrium through a system of first order differential equations in \( K(t), h(t), C(t) \) and \( V(t) \). Using Eqs. (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14) and (15), we get
\[
\dot{K}(t) = \beta \alpha A \left[ K(t)/h(t) \right] \left[ z_1(V(t)) \right]^{\beta - 1} - C(t) - \delta K(t), \quad (16a)
\]
\[
\dot{h}(t) = \gamma B^\alpha \Psi^\beta \left[ 1 - (1-t_k) \beta A \right] \left[ z_1(V(t)) \right]^{\beta - 1} h(t) - \eta h(t), \quad (16b)
\]
\[
\dot{C}(t) = (C(t)/\sigma) \left[ (1-t_k) \beta A \right] z_1(V(t))^{\beta - 1} - \delta - \rho, \quad (16c)
\]
\[
\dot{V}(t) = \left( (1-t_k) \beta A \right) z_1(V(t))^{\beta - 1} - \delta + \eta V(t) - (1-t_k)(1-\beta)A \left[ z_1(V(t)) \right]^{\beta}. \quad (16d)
\]

At a balanced growth path (BGP, henceforth) the stock of physical capital, the stock of human capital and consumption grow at constant rates, and the factor shares in the technologies are constant. Hence, we see from (16c) and (14a) that \( V(t) \) is also constant along a BGP.
Furthermore, given the definition of \( z_i(t) \), the previous property implies that the growth rate of \( K(t) \) in a BGP must therefore be the same as \( h(t) \) since \( u(t) \) and \( v(t) \) are constant. Finally, dividing (16a) by \( K(t) \), we get that \( K(t) \) must grow at the same rate as \( C(t) \) along a BGP. As is customary in the economic growth literature, we shall focus on cases in which the economy exhibits interior BGPs, along which prices and resource allocations are constant and strictly positive. The next proposition establishes the existence and the uniqueness of an interior BGP.

**PROPOSITION 1.** Consider the economy described by Eqs. (4), (5), (6), (7), (8), (9), (10), (11) and (13). There exists at most an interior BGP equilibrium.

**Proof.** After making \( \dot{V}(t) = 0 \), and using (14a), Eq. (16d) can be written as

\[
\delta - \eta = (1 - t_h)\beta\alpha\left(\frac{\beta}{1 - \beta}\right)\Omega^\beta - \left(1 - t_h\right)(1 - \beta)\alpha\theta\left(\frac{\beta}{1 - \beta}\right)\Omega^\beta, \tag{17}
\]

where \( V^* \) denotes the stationary value of \( V(t) \). The right-hand side of the previous equation is a monotonic function of \( V^* \). Therefore, there is at most one value of \( V(t) \) which generates a BGP equilibrium. Furthermore, given relationships (14) and (15) one can prove from (16) that there is a one-to-one correspondence between \( V(t) \) and the growth rate of \( K(t), h(t) \) and \( C(t) \). Q.E.D.

The following proposition establishes the local stability property of the interior BGP:

**PROPOSITION 2.** Consider the economy described by Proposition 1. Consider the following subsets of \( \Pi \):

\[
\Pi_1 = \{ \pi \in \Pi | \beta > \alpha \},
\]

\[
\Pi_2 = \{ \pi \in \Pi | \alpha \theta < \beta < \alpha \text{ and } \beta[(1 - \theta)(1 - s_E) + \theta(1 - t_h - s_u)] < \alpha \theta (1 - t_h - s_u) \},
\]

\[
\Pi_3 = \{ \pi \in \Pi | \alpha \theta < \beta < \alpha \text{ and } \beta[(1 - \theta)(1 - s_E) + \theta(1 - t_h - s_u)] > \alpha \theta (1 - t_h - s_u) \},
\]

\[
\Pi_4 = \{ \pi \in \Pi | \beta < \alpha \theta \text{ and } \beta[(1 - \theta)(1 - s_E) + \theta(1 - t_h - s_u)] < \alpha \theta (1 - t_h - s_u) \},
\]

\[
\Pi_5 = \{ \pi \in \Pi | \beta < \alpha \theta \text{ and } \beta[(1 - \theta)(1 - s_E) + \theta(1 - t_h - s_u)] > \alpha \theta (1 - t_h - s_u) \}.
\]

(i) If \( \pi \in \Pi_1 \cup \Pi_3 \cup \Pi_4 \), then the equilibrium is locally saddle-path stable.
(ii) If \( \pi \in \Pi_2 \), then the equilibrium is locally indeterminate of degree two, i.e., there is locally a continuum of equilibria.

(iii) If \( \pi \in \Pi_5 \), then the equilibrium is locally unstable.

**Proof.** See the appendix. Q.E.D.

Figure 1 summarizes the previous result. The figure draws the different regions characterizing the local stability properties of the BGP in the \( (\theta, \alpha) \) space while keeping the other parameters constant. We observe that in our model the local saddle-path stability of the BGP equilibrium does not hold for whole space of parameters. The previous equilibrium can also be either locally indeterminate or locally unstable. An interesting point of our stability result is that the existence of multiple equilibria requires that the government implement a specific combination of fiscal instruments. In absence of fiscal policy, the two curves defining the regions \( \Pi_2 \) and \( \Pi_5 \) coincide and, thus, these regions disappear. Furthermore, either increases in \( s_E \) or reductions in \( t_h \) and \( s_w \) raise the region of indeterminacy and reduce the region of instability.

[Insert Figure 1 around here]

However, given this necessary intervention of the fiscal authority, the key for our result is the assumption that human capital accumulation uses a technology that combines intermediate market goods and non-working time. One can note that under either \( \theta = 0 \) or \( \theta = 1 \) the subsets of parameters \( \Pi_2 \) and \( \Pi_5 \) are both empty. Moreover, this is also the case when \( \alpha = \beta \). In these extreme cases, the presence of growth-distorting fiscal policies never affects the stability property of the BGP equilibrium.\(^8\)

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\(^8\) Ortigueira (1998) proves that the presence of capital income taxation does not alter the saddle-path stability of the model with pure non-market human accumulation. Mino (1996) shows the same as the previous author for the model with pure market human capital accumulation. Finally, for the latter model, Alonso-Carrera (2000) proves that a subsidy to the purchases of new human capital does not affect the saddle-path stability result.
4. Discussion

What is the economic mechanism underlying the equilibrium dynamics described in the previous section? Since all prices are determined by the discounted marginal value of human capital alone (see (9) and (14)), and given the block-recursive structure of the dynamic system (16), equilibrium prices are determined independently of quantities. Furthermore, recall that the dynamic behavior of those variables is dictated by the ranking of factor intensities across sectors. On the one hand, the adjustment process of equilibrium prices is stable if the production of intermediate goods and the activities accumulating human capital jointly employ a smaller relative quantity of physical capital. Otherwise, the previous process is unstable. This is because of the Stilper-Samuelson effect. If the production of new human capital is labor intensive, a sudden increase in the price of human capital raises the wage rate and, thus, the intertemporal no-arbitrage condition for the selection between physical and human capital investment will drive the relative price of human capital down. On the other hand, when physical capital commands a higher relative cost share in the production of new human capital, the quantity process is saddle-path stable. In the opposite case, this process is unstable. This stability property of the quantity process follows from the Rybczynski effect. If the production of new human capital is labor intensive, a sudden increase in the stock of human capital results in an increase in the production of new human capital and a decrease in the production of physical goods.

Therefore, regardless of the factor intensities of each sector, the BGP is locally saddle-path stable if and only if the rankings of factor intensities in the value and physical sense coincide. However, the taxation policy introduces a wedge between both rankings of factor intensities, so that the distortion from the taxation policy governs the stability property of the economic system. If the two factor intensity rankings still coincide, the BGP is locally saddle-path stable. By contrast, if the tax distortion is sufficiently high, the two rankings of factor intensities are no consistent and, thus, the BGP is either locally indeterminate or unstable.
To interpret the stability results given by Proposition 2 in terms of this reversal in the ranking of factor intensities, we substitute (3) in (4). Hence, using (A.3) from the appendix, and after a simple manipulation, we get the following reduced-form technology capturing the influence of the technologies for producing intermediate goods and for accumulating human capital:

\[
\hat{h}(t) = \gamma B^\alpha \psi^{\theta(1-\alpha)} \left[ (1 - v(t))K(t) \right]^{\alpha\theta} \left[ (1 - l(t))h(t) \right]^{1-\alpha\theta} - \eta h(t) .
\] (17)

Note that the physical capital intensity in terms of factor quantities ratios for the production of new human capital is given by \( \alpha \theta \), whereas \( \alpha \theta(1-t_h-s_u) \) is the intensity in terms of factor cost shares. Similarly, while \( \beta \) is the physical capital intensity in terms of factor quantities ratios for the physical goods sector, \( \beta \left[ (1-\theta)(1-s_E) + \theta(1-t_h-s_u) \right] \) is the intensity in terms of factor cost shares. If \( \alpha \theta < \beta \) and \( \beta \left[ (1-\theta)(1-s_E) + \theta(1-t_h-s_u) \right] < \alpha \theta(1-t_h-s_u) \), then the process governing the equilibrium value of human capital is stable and the quantity process is saddle-path stable. In this case, the BGP is therefore locally indeterminate. The previous two dynamic processes are both unstable if \( \alpha \theta > \beta \) and \( \beta \left[ (1-\theta)(1-s_E) + \theta(1-t_h-s_u) \right] > \alpha \theta(1-t_h-s_u) \). Otherwise, the BGP is locally saddle-path stable since the two rankings of factor intensities coincides and, thus, an unstable process in prices or quantities is offset by adjustment in the other variable.

The unstable and the indeterminate equilibrium dynamics emerge in our model under conditions for the reversal in the factor intensity ranking that are the opposite of those found by Bond, Wang and Yip (1996) in the case in which the accumulation of human capital is a pure market-activity, i.e., \( \theta = 1 \). These authors obtain indeterminacy when the sector producing human capital is labor intensive in terms of factor cost shares but capital intensive in terms of factor quantity ratios. By contrast, we do not need a technology for human capital accumulation that is more physical capital intensive in terms of factor quantity ratios than that used for producing physical goods. The required condition for our result is that the sector producing intermediate goods would be more physical capital intensive than the physical good sector, i.e., \( \alpha > \beta \). However, the reduced-form technology for producing new human capital must be less physical capital intensive than that for producing physical goods, i.e., \( \alpha \theta < \beta \). Although it would remain to measure the plausibility of our
technological restriction, this condition seems to be empirically less strong than the Bond, Wang and Yip's one. While the average share of physical capital in final output is usually calibrated around 0.4, Perli and Sakellaris (1998) estimate this share in educational output at a range between 0.11 and 0.17.

Relating with the previous discussion, we should also verify whether indeterminacy appears for plausible values of parameters. We next present a numerical example to illustrate that the combinations of parameters that lead to indeterminacy are empirically plausible. Suppose, for example, that  

\[ A = 1, \quad B = 1, \quad \gamma = 0.18, \quad \rho = 0.01, \quad \sigma = 1.5, \quad \delta = 0.1, \quad \eta = 0.1, \quad \alpha = 0.53, \quad \beta = 0.36, \quad \theta = 0.64, \quad t_k = 0.21, \quad t_h = 0.31, \quad s_e = 0.5 \text{ and } s_w = 0.08. \]

The values of \( A, B, \delta, \eta, \beta, t_k \) and \( t_h \) agree with those generally invoked in the economic literature (see, e.g., Jones, Manuelli and Rossi, 1993). OECD (1992) estimates for US that the public expenditure in secondary and higher levels of education amounts to 50% of total education expenditure in these levels, whereas the fellowships represent around 8% of the opportunity cost associated to these levels of education.\(^9\) These facts justify our choice of the values of \( s_e \) and \( s_w \). By recalling the estimation of Perli and Sakellaris (1998) given in Section 1, the value of \( \theta \) was chosen so that the opportunity cost of accumulating human capital amounts to 31% of total educational cost along the BGP. We have jointly obtained the values of \( \gamma, \rho \) and \( \sigma \) by imposing that the after tax, net interest rate and the stationary growth rate equal to 0.04 and 0.02 at the BGP, respectively. Finally, the value of \( \alpha \) was fixed in order to force the vector of parameters \( \pi \) to lie in \( \Pi_2 \). Although the later value could seem a little large, the lack of reliable data prevents us from addressing a closed conclusion about the plausibility.

The possibility of multiple equilibria stands a chance to explain unpredictable differences in the growth patterns followed by countries with identical economic situations of departure. In particular, the present model predicts that identical economies with a specific fiscal policy structure may follow different equilibrium paths to reach a common long-run growth rate, so that each country’s per

\(^9\) In this example, we identify, as is usual, the production of new human capital as education. We then ignore the investment in other concepts of human capital as, for instance, health.
capita income permanently diverges one from the other. Evidently, in our model, as in any endogenous growth model, fiscal policy can be a source of differences in both the long-run growth rate and in the growth patterns followed by otherwise identical economies. However, the present paper also predicts that two identical countries, even with the same fiscal policy structure, may follow different equilibrium paths. Intuitively, local indeterminacy of the equilibrium means that the expectations determine the equilibrium path since in this case the initial level of consumption and the allocation of time between different activities are freely chosen. Therefore, our economy becomes vulnerable to sunspot equilibria due to the presence of fiscal policy. In order words, indeterminacy is also able to account for business cycles without having to rely on shocks to fundamentals.10

Evidently, when multiple equilibrium paths exist, they could be ranked by following the welfare criterion. However, the question is how individuals agree to select the equilibrium path generating the highest welfare. Equilibrium indeterminacy implies a problem of coordination failure. How do individuals coordinate their expectations for selecting an equilibrium path? This problem opens an interesting line for future research. We should look for alternative mechanisms of equilibrium selection, i.e., mechanisms through which individuals can coordinate their expectations.

APPENDIX

Reducing the explanatory variables of the model

We will next solve for the factor intensities in each sector, the relative price and shares of both types of capitals in each technology as functions of $K(t)$, $h(t)$ and $V(t)$. First, diving (9a) by (9b), we have that

$$z_2(t) = z_1(t)\left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right).$$

10 The dependence of the stability property on the parameter structure may also lead to the existence of limit cycles. To check this point, we should follow a bifurcation analysis, which we have omitted since this is outside the scope of this paper.
Plugging the previous equation in (9a) and (9b), we respectively get

\[
\begin{align*}
  z_1(t) &= \left[ \left( \frac{\alpha B}{\beta A} \left( \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \right) \right)^{\alpha-\beta} \right]^p(t)^{\beta-\alpha}, \\
  z_2(t) &= \left[ \left( \frac{\alpha B}{\beta A} \left( \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \right) \right)^{\beta-\alpha} \right]^p(t)^{\beta-\alpha}.
\end{align*}
\]

Introducing (A.1) and (A.2) in the non-arbitrage condition (11), we directly derive the relationship between \( p(t) \) and \( V(t) \) given by (14c). Moreover, Eqs. (14a) and (14b) are obtained by inserting (14c) in (A.1) and (A.2), respectively.

Second, we will use (14) to derive the allocations of both types of capitals to each sector as functions of \( K(t) \), \( h(t) \) and \( V(t) \). Dividing (13) by (11), and using (8) and (9), we get

\[
\frac{l(t) - u(t)}{1 - l(t)} = \left( \frac{1 - t_h - s_w}{1 - s_t} \right) \left( \frac{\theta(1-\alpha)}{1-\theta} \right).
\]

(A.3)

Moreover, full employment requires that

\[
u(t)z_1(t) + (l(t) - u(t))z_2(t) = K(t)/h(t).\]

(A.4)

Solving the system composed of Eqs. (A.3) and (A.4) for \( l(t) \) and \( u(t) \), and rewriting the solution with (14), we derive (15a) and (15b). Expression (15c) is directly obtained from (15b) by noting that \( z_1(t) = [v(t)K(t)]/[u(t)h(t)] \).

**Proof of Proposition 2**

We first introduce two new variables: \( z(t) = K(t)/h(t) \) and \( x(t) = C(t)/K(t) \). Using dynamic system (16), we derive the dynamic equations describing the law of motion of the previous variables

\[
\begin{align*}
  \dot{z}(t) &= z(t) \left\{ A u (z(t), V(t)) \right\} \left[ z_1(V(t)) \right]^{\beta} - x(t) - \delta \\
  &\quad - \gamma B^\phi \Psi^\phi \left[ 1 - l(z(t), V(t)) \right] \left[ z_2(V(t)) \right]^{\alpha} + \eta, \\
  \dot{x}(t) &= x(t) \left[ (1/\sigma) \left( 1 - t_k \right) B [z_1(V(t))]^{\beta-1} - \delta - \rho \right]
\end{align*}
\]

(A.5)
\[ -A u(z(t), V(t)) z(t)^{-1} (z_1(V(t)))^\beta + x(t) + \delta \]  
\hfill (A.6)

Since the initial value of \( z \) is fully defined by \( K_0 \) and \( h_0 \), the system of differential equations (A.5), (A.6) and (16d) fully summarizes the competitive equilibrium paths. To establish the local stability properties of the BGP equilibrium, we linearize this reduced system around the stationary values of \( z(t), x(t) \) and \( V(t) \), which will be denoted by \( z^*, x^* \) and \( V^* \). This linearized system is as follows:

\[
\begin{bmatrix}
\dot{z}(t) \\
\dot{x}(t) \\
\dot{V}(t)
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
0 & 0 & a_{33}
\end{bmatrix}
\begin{bmatrix}
z(t) - z^* \\
x(t) - x^* \\
V(t) - V^*
\end{bmatrix},
\hfill (A.7)
\]

where the elements \( a_{ij} \) are obtained from differentiating (A.5), (A.6) and (16d). The local stability is determined by the sign of the eigenvalues associated with the Jacobian matrix (A.7). These eigenvalues, which will be denoted by \( \xi_i \), are the roots of the following characteristic polynomial:

\[
Q(\xi) = \left( a_{33} - \xi \right) \left\{ \xi^2 - (a_{11} + a_{22}) \xi + (a_{11}a_{22} - a_{12}a_{21}) \right\},
\hfill (A.8)
\]

where

\[
a_{11} = z^* \left\{ -\frac{a_{21}}{x^*} + \frac{\gamma B^\theta \Psi^\theta z_2(V^*)^{\alpha \theta}}{z_1(V^*) + \Psi(z_1(V^*) - z_2(V^*))} \right\},
\]

\[
a_{12} = -z^* ,
\]

\[
a_{21} = -\frac{A \Psi x^* z_2(V^*) z_1(V^*)^\beta (z^*)^{-2}}{z_1(V^*) + \Psi(z_1(V^*) - z_2(V^*))},
\]

\[
a_{22} = x^* ,
\]

\[
a_{33} = -\left( \frac{\alpha \theta}{\beta - \alpha \theta} \right) (1-t_k) \beta A z_1(V^*)^{\beta - 1} - \delta + \eta \left( \frac{1-\beta}{\beta - \alpha \theta} \right) (1-t_k) \beta A z_1(V^*)^{\beta - 1}.
\]

Using (14) and (15), we obtain that

\[
\text{Sign} \{a_{11}\} = \text{Sign} \left\{ z_1(V^*) + \Psi(z_1(V^*) - z_2(V^*)) \right\},
\]

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\[ \text{Sign}\{a_{31}\} = \text{Sign}\{-1a_{11}\}, \]

\[ \text{Sign}\{a_{33}\} = \text{Sign}\{-1(\beta - \alpha)\}. \]

Moreover, using (14) and (15), and after some algebra, we get that,

\[
\text{Sign}\left\{ \left[z_1(V^+) + \Psi(z_1(V^+) - z_2(V^+)) \right] \right\}
\]

\[= \text{Sign}\left\{ \beta \left(1 - s_E \right)(1 - \theta) + \theta \left(\beta - \alpha\right)(1 - t_h - s_w) \right\}, \]

At this point, one can easily prove Proposition 2. First, note that one of the roots of characteristic polynomial (A.8) is equal to \(a_{33}\). Moreover, we observe that the expression inside brackets in (A.8) is not more than the characteristic polynomial associated with the sub-matrix obtained from dropping the last row and the last column of the Jacobian matrix (A.7). The trace and the determinant of this sub-matrix are equal to \((a_{11} + a_{22})\) and \((a_{11}a_{22} - a_{12}a_{21})\), respectively. The sign of the eigenvalues of this sub-matrix are easily determined by noting that the trace and the determinant are respectively given by the sum and the determinant of these eigenvalues. If \(\beta \left(1 - s_E \right)(1 - \theta) + \theta \left(\beta - \alpha\right)(1 - t_h - s_w) > 0\), then they are both positive. Otherwise, these two eigenvalues are of the opposite sign. Thus, the result directly follows.

References


Fig. 1. Local stability properties of the BGP

\[ \alpha = \beta \left[ 1 + \frac{1 - \theta}{\theta} \frac{1 - s_x}{1 - r_x - s_x} \right] \]